

Reactive Power Cost Analysis with Generators and Condensers in Deregulated Power System

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Abstract. Reactive power procurement has been identified as an important ancillary service. The cost characteristics must be able to recover the lost opportunity cost component of generator. The main contribution of the paper is: (i) Cost characteristics determination for synchronous generator based on capability chart, (ii) location of hydro condenser based on losses reduction and voltage profile improvement, (iii) cost analysis of reactive power supported by synchronous generator and condenser, (iv) impact of hydro condenser on reactive support. The reactive power cost function has been obtained for each generator and have been utilized to find the cost of reactive power support. The results have also been compared with the existing methods. The reactive power cost analysis has been carried out for IEEE 14 bus test system considering hydro condenser and its impact on the reactive power supply from the other generators and thermal condenser.

Keywords: Capability curve, generation costs, reactive power, cost of reactive power.

1. Introduction

The ISO procure reactive power as an ancillary service from the synchronous generators and condensers to maintain voltage profile and thereby the security of the system. Since cost is involved to procure reactive reserve services, the service providers should be remunerated. The lost opportunity cost has to be recovered to make sustaining the ancillary services providers in the competitive electricity markets. Therefore, the operation and cost allocation of reactive power has become essential for proper management of generation and/or transmission companies which is essential responsibilities of the ISO [1]. For the recovery of the cost of reactive power, most of the utilities charge industrial consumers based on KVA demand and penalize the utilities for the poor power factor. This fixed cost recovery of reactive power is insufficient and in the competitive markets may not provide accurate price signal for the ancillary providers [2]. Looking into the limitations of a reactive power price to be charged from the consumers based on power factor penalties authors suggested the use of economic principles based on marginal theory [3-4]. However, the prices of reactive power based on the marginal price theory represent a small portion of the actual reactive power price [5-7]. Reactive power has typical characteristics due to reactive power generation equipment and local characteristics of reactive power of loads and transmission system behavior; therefore, it is difficult to access its cost characteristics [8]. For the assessment of reactive power cost, many authors proposed different techniques [10-18]. Dandhachi and Choi et al. proposed real time pricing method for reactive power using OPF method [9-11]. The reactive power marginal price is typically less than 1% of the active power marginal price and depends strongly on the network constraints as proposed by Hao and Papalexopoulos [10]. Gill GB et al. proposed a conceptual framework for remuneration and charging of reactive power cost [12]. A cost based reactive power dispatch method is proposed for reactive power cost calculation and explicit and implicit cost of reactive support from generator and transmission system is also analyzed [13]. However the method of opportunity cost calculation of reactive support from generators is quite complex. A summary of

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different methods for reactive power pricing has been discussed using different objective functions for calculation of marginal prices of reactive power using optimal power flow based approach in [14]. Ketabi et al. in [15] proposed a pricing technique based on minimization of the generator active and reactive power production and capacitor bank costs using the ant colony algorithm. Ro has presented the reactive charging scheme composed of recovering capital cost and operational cost [16]. A new approach for costing of reactive power was proposed in [17]. The proper recovery of reactive power cost with help sustaining the ancillary service providers in the electricity markets. The cost must be decided based on the capability limits of the generators at each operating point due to the load variations and power factor variations.

The main contribution of the paper is: (i) Cost characteristics determination for synchronous generator based on capability chart, (ii) location of hydro condenser based on losses reduction and voltage profile improvement, (iii) cost analysis of reactive power supported by synchronous generator and condenser, (iv) impact of hydro condenser on reactive support. A case study of the IEEE 14-bus system has been presented in this work. Different operating conditions have been considered to obtain the reactive support and the cost function for reactive power has been obtained using the curve fitting technique. The cost curves obtained are utilized in IEEE bus test system to obtain the cost of reactive support based on operating point in the system.

2. Reactive Power Generation Cost

2.1 Cost model of Synchronous generator reactive power supply

A curve for reactive power generation in the generator and the condenser mode is obtained and is shown in Fig. 1. Figure 1 show that the condenser mode region is much smaller than the generator mode region. Region (i) and (iii) supply reactive power to the system in generator and condenser mode respectively. Region (ii) and (iv) absorb reactive power from the system in generator and condenser mode respectively. Negative P indicates that active power is absorbed from the grid to keep machine running in condenser mode and for supplying the losses.

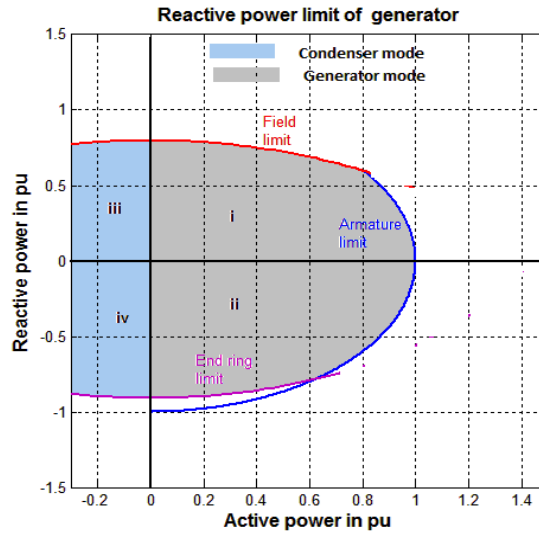


Fig.1: Reactive power generation in generator and condenser mode.

The lost opportunity cost can be determined above the rated reactive power requirements and below the maximum limit reactive power generation from the generators. Therefore, the reduction in active power has to be calculated by the field limit equation. From field current limit equations,

$$P_t = \frac{|E_t| |E_q|}{X_s} \sin \delta \quad (1)$$

$$Q_i = \frac{|E_t| |E_q|}{X_s} \cos \delta - \frac{|E_t|^2}{X_s} \quad (2)$$

$$\cos \delta = \left(Q_i + \frac{|E_t|^2}{X_s} \right) \cdot \frac{X_s}{|E_t| |E_q|} \quad (3)$$

The real power output limited by the field limits can thus be obtained as:

$$P_i = \sqrt{\left(\frac{|E_t||E_q|}{X_s}\right)^2 - \left(Q_i + \frac{|E_t|^2}{X_s}\right)^2} \quad (4)$$

Q will be maximum when $P_i=0$

$$Q_{\max} = \frac{|E_t||E_q|}{X_s} - \frac{|E_t|^2}{X_s} \quad (5)$$

$$Q_{\text{rated}} = S \cdot \sin(\Phi) \quad (6)$$

The change in the real power can be obtained from its rated value to any operating value as:

$$\nabla P_i = P_{\text{rated}} - P_i \quad (7)$$

The cost of decrease in the real power output can be obtained using the fuel cost function.

$$\text{Cost}(\nabla P_i) = a(\nabla P_i)^2 + b(\nabla P_i) + c \quad (8)$$

This cost of reduced active power shall be recovered from the reactive power support obtained from the generators as:

$$\text{Cost}(Q_i) = \begin{cases} 0 & \text{if } Q_i \geq Q_{\text{rated}} \\ \text{Cost}(\nabla P_i) = \text{Cost}(P_{\text{rated}}) - \text{Cost}(P_i) & \text{if } Q_{\text{rated}} \leq Q_i \leq Q_{\max} \end{cases} \quad (9)$$

The cost of reactive power following the capability chart of the generators can thus be plotted based on the reactive power support provided by the generators. Based on the data obtained, plot the graph Q vs Cost (Q) and by using of curve fitting toolbox in MATLAB, quadratic equation can be obtained for the reactive power cost can then be expressed as:

$$\text{Cost}(Q) = a_q Q^2 + b_q Q + c_q \quad (10)$$

2.2 Reactive power generation cost for the thermal synchronous condenser

Based on the cost data as given in [13][10], the reactive power generation cost for condenser is calculated as:

Total installation cost (TC)=1000000 \$/MW, Installation capacity (MW)=60 MW, Availability factor (AF)=0.25, Load factor (LF)=0.6, Rated power factor (pf)=0.9, life of the plant considered (N)=30 year

$$CF_i = \frac{TC}{IC \cdot LF \cdot AF \cdot 8760 \cdot N} \tan(\cos^{-1}(\text{pf})) \cdot 12.28 \frac{\$}{\text{MVA} \cdot \text{hr}}, \text{ Energy consumed (Pg)}=0.03 \cdot 60 = 1.8 \text{ MW, Cost (Pg)}=a \cdot \text{Pg}^2 + b \cdot \text{Pg} + c = 109.4262 \text{ \$/hr, Cost (Q)}=12.28 \cdot Q + 109.42 \text{ \$/hr}$$

2.3 Reactive power generation cost of the hydro synchronous condenser [10]

Based on the cost data as given in [13][10], the reactive power generation cost for condenser is calculated as:

In this, cost components in hydro condenser such as cost of water lost (C_w) and cost of wear and tear of mechanical system (C_t) are also included. The minimum number of changeover is taken as 2.

Total installation cost, TC=127300000 \$, installation capacity (MW)=250MW, Availability factor (AF)=0.25 Load factor (LF)=0.6, Rated power factor (pf)=0.9, life of the plant considered (N)=35 year

$$CF_i = \frac{TC}{IC \cdot LF \cdot AF \cdot 8760 \cdot N} \tan(\cos^{-1}(\text{pf})) = 5.36 \frac{\$}{\text{MVA} \cdot \text{hr}}, \text{ Energy consume (Pg)}=0.03 \cdot 250 = 7.5 \text{ MW, Cost (Pg)}=a \cdot \text{Pg}^2 + b \cdot \text{Pg} + c = 123.66 \text{ \$/hr, Changeover cost (Cc)}=(C_w + C_t) \cdot \text{no. of change over} = (0.45 + 4.2) \cdot 2 = 9.3 \text{ \$}$$

$$\text{Cost}(Q) = 5.36 \cdot Q + 123.66 + 9.3 = 5.36 \cdot Q + 132.96 \text{ \$/hr}$$

2.4 Cost Characteristics Curves for Generators in IEEE 14 Bus test systems

The cost of real and reactive power has been obtained for 500MVA and 60 MVA generators in IEEE 14 bus test system. From figures it is observed that the lost opportunity cost can be recovered at every operating point on the capability curve of the generator. The active power and the reactive power cost curve obtained for 500 MVA generators is shown in Fig. 2 and 3. The cost of active and reactive powers is also obtained for 60 MVA generators. From figure, it is observed that the cost of loss of opportunity cost is recovered at any operating point on the capability curves for generators. The cost coefficients obtained for different rating generators are given in Table 1.

Table 1. Reactive power cost coefficients obtained based on capability curves

| Generator | S (MVA) | a_q \$/MWhr ² | b_q \$/MWhr | c_q | Q(MVAr) |
|-----------|---------|----------------------------|---------------|----------|---------------------------|
| G1 | 500 | 0 | 0 | 0 | $Q \leq Q_{rated}$ |
| | | 0.0385 | -9.1451 | 607.9682 | $Q_{rated} < Q < Q_{max}$ |
| G2 | 60 | 0 | 0 | 0 | $Q \leq Q_{rated}$ |
| | | 0.5180 | -19.1597 | 184.8086 | $Q_{rated} < Q < Q_{max}$ |
| G3 | 60 | 0 | 0 | 0 | $Q \leq Q_{rated}$ |
| | | 0.5736 | -21.2958 | 206.1758 | $Q_{rated} < Q < Q_{max}$ |

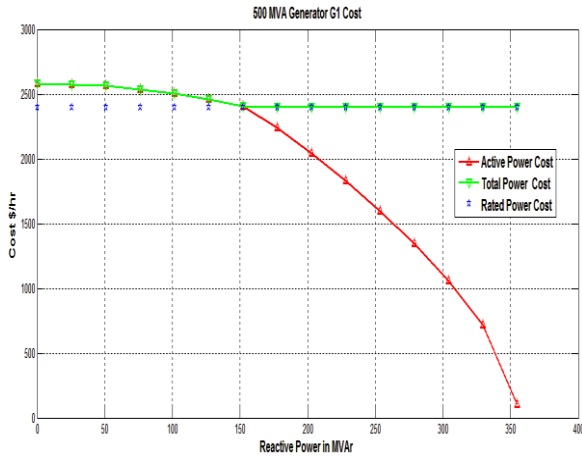


Fig. 2: Active power cost at any operating point and rated active power cost.

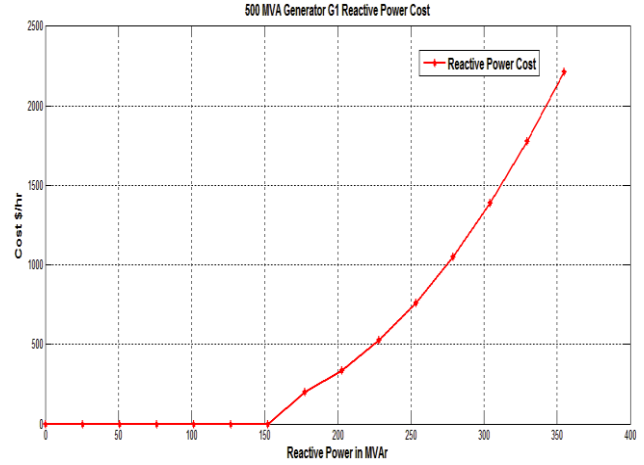


Fig. 3: 500 MVA Generator G1 Reactive power cost.

3. Case study on IEEE 14 bus system

In this study, the reactive power price has been obtained for IEEE 14 bus system. The 14 bus system has two generators G1 and G2 at bus 1 and 2 respectively. Synchronous condenser is also connected at load bus 3 in the system. Another hydro condenser have been taken and its placement has been decided based on the reactive load at a particular bus and voltage dip at the buses due to the reactive power load increase. Based on the cost characteristics of reactive power obtained for 500 and 60 MVA generators and cost of reactive power support from synchronous condensers, the reactive power price has been obtained at different operating point in IEEE 14 bus system taking the reactive power support from generators and synchronous condensers. The impact of load increase has been obtained on the voltage profile and reactive power requirement in the system. The placement of synchronous condensers has been obtained based on the voltage dip at the respective buses due to the load increase and the reactive load requirement at the respective buses.

3.1 Results and Discussions

The placement of hydro condenser has been considered on the two bases:

(i) Maximum load buses: On the basis of load from table, three busses bus 2, bus3 and bus9 has chosen as candidate bus for placement of hydro condenser, (ii) Maximum voltage dips at the respective buses.

The two different cases have been categorized as:

Case1: Case1a: hydro condenser at bus 3 as a 1st maximum reactive power load bus.

Case1b: hydro condenser at bus 9 as a 2nd maximum reactive power load bus.

Case1c: hydro condenser at bus 2 as a 3rd maximum reactive power load bus.

Case2: Case2a hydro condenser at bus 14 based on 1st maximum voltage dip

Case2b hydro condenser at bus 13 based on 2nd maximum voltage dip

Case2c hydro condenser at bus 10 based on 3rd maximum voltage dip

Running load flow, the voltage profile with increasing reactive power load at all buses by multiplying factor λ from 1 to 1.5 has been obtained and is shown in Fig.4. It can be observed that when reactive power load at all buses is increased by multiplying factor λ from 1 to 1.5, the voltage dip at bus 14 is lower than 0.95 p.u. at $\lambda=1.2$. With further increase in the $\lambda=1.3$, two buses 13 and 14 are observed with voltage lower than 0.95 p.u. At $\lambda=1.4$ buses from 9 to 14 are having lower voltage below 0.95 p.u. and at $\lambda=1.5$, buses from 6 onwards are having lower voltage. Buses 10, 13 and 14 are observed with lower voltages and these

three buses are considered as a candidate bus for the placement of the hydro condenser. After the placing the hydro condenser at different buses by considering the above two cases, the real and reactive power loss has been obtained and is shown in Figs. 5 and 6. It is observed that with reactive power load increase for all the cases, the real and reactive power loss increases. At base case active and reactive power losses are higher and for case1 and case2, the losses reduces and is observed lower for case2a. It is observed from the figures that the real power loss is lower with case2a when hydro condenser is placed at bus 14. The reactive power loss obtained is also lower with case2a when hydro condenser is placed at bus 14. Due to placement of the hydro condenser at different buses active and reactive power losses has decreased. Based on the losses profile, the hydro condenser has been placed on bus 14. In case1 with all sub cases, the losses are observed higher. This shows that placing the reactive source only considering reactive load is not sufficient, but the losses profile shall also be considered for reactive source placement. From losses, the placement of the hydro condenser at bus14 is best place for the placement of hydro condenser. Now after placing hydro condenser at bus 14, the voltage profile has been obtained and is shown in Fig. 7 for the load increase at all the buses with multiplying factor λ varying from base case to $\lambda=1.2$ to 1.5. It is observed that the voltage at all the buses is above 0.95 p.u. at each value of reactive power load increase. After the placement of hydro condenser at bus 14, it is observed that the voltage profile is better than without hydro condenser. And we see that the bus voltages within the 0.98 to 1.06. At base case no hydro condenser placement is considered because the bus voltages are within the permissible voltage level i.e 1.05 p.u. and 0.9 5p.u.

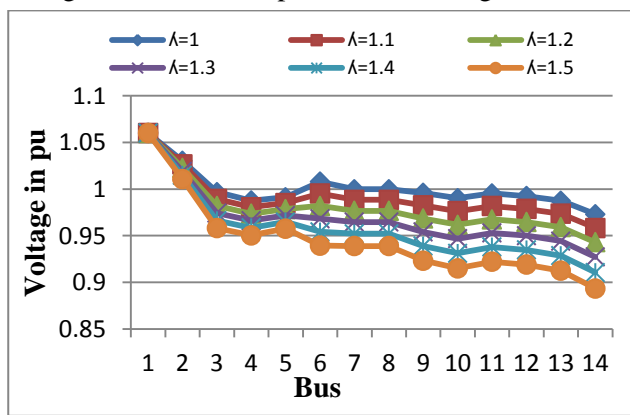


Fig. 4: Voltage profile at base case without hydro condenser

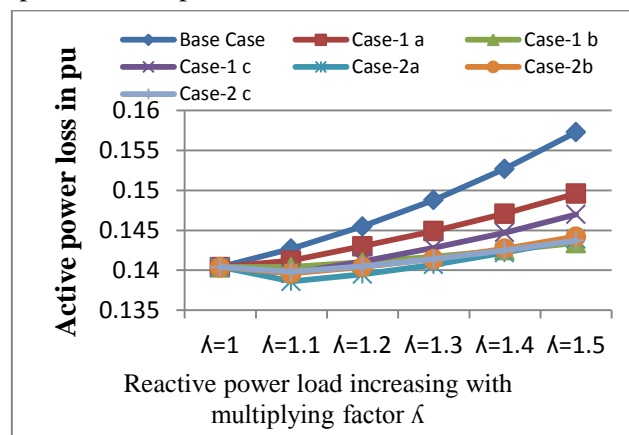


Fig. 5: Active power loss due to increase in reactive power load

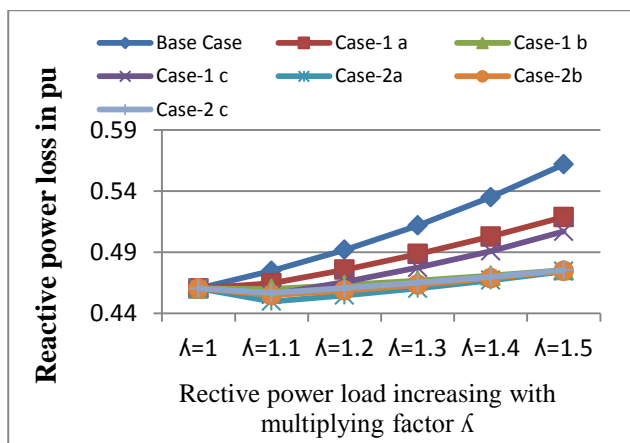


Fig. 6: Reactive power loss due to increase in reactive power load

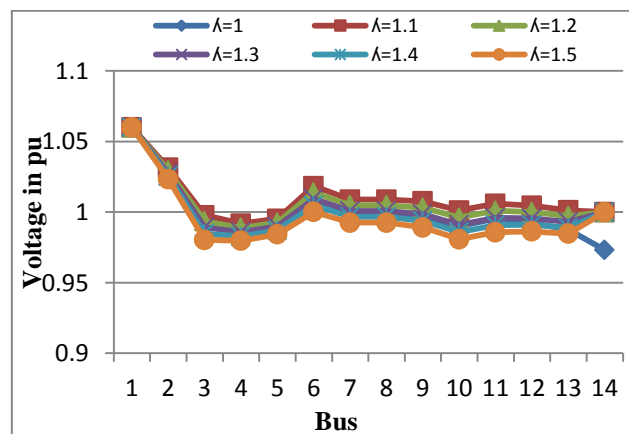


Fig. 7: Voltage profile after placing hydro condenser at bus14

Without hydro condenser, the reactive power generation is at its maximum value for G2 and thermal condenser at bus 3. With the reactive power load increase, the additional reactive power requirement is met by the synchronous generator at bus 1. Thus, the reactive power generation reaches to 0.75 p.u. at the $\lambda = 1.5$. The reactive power requirement supplied by the G2 and thermal condenser are at its limits. With the addition of the hydro condenser, the reactive power requirement from G1 reduces with reactive load increase and the hydro condenser supplies the reactive power with increase in the reactive power load. The reactive power

supplied by the generators G1, G2 and the thermal condenser is shown in Fig. 8. Figure 9 shows that with the variation of reactive load, generator G2 and thermal condenser reactive power generation has reached maximum limit. Rest reactive power demand has been fulfilled by the generator G1. Thus, generator G1 is more stressed due to supply of reactive power and due to the reactive power supply by the generator G1 to the loads, losses in the system increases due to the higher reactive power flow. With the placement of hydro condenser at bus 14 and with increasing in load by multiplying factor λ , the reactive power generation by generator G2 and thermal condenser reactive power generation are at their maximum limiting value and the rest of reactive power demand has been taken care by generator G1 and hydro condenser. Due to placement of the hydro condenser, the reactive power supplied by the generator G1 reduces and thus the losses in the system also reduces. The hydro condenser supplies the reactive power with the increasing reactive load and the losses are minimum, voltage profile has also improved with the support of reactive power. The reactive power supplied by the generator G1 is 0.4 p.u. and hydro condenser supply the reactive power of 0.24 p.u. The reactive power of 0.75 p.u. provided by G1 has now reduced to 0.4 p.u. as the source of reactive power is available near load bus and reactive power consumption of line has reduces thereby the losses reduces and voltage profile has also become better. The reactive power generation and cost at each load increase for case1 and case2 are also given in Table 2.

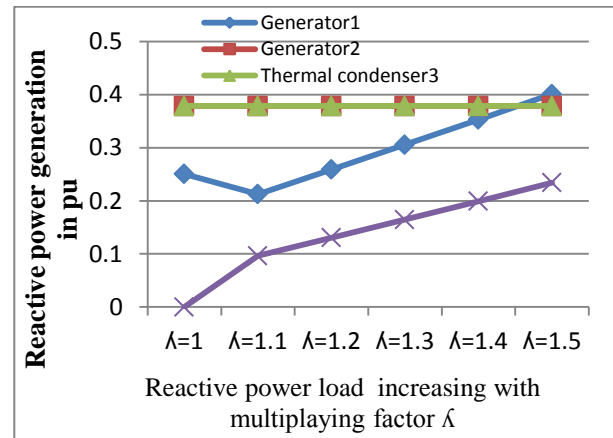
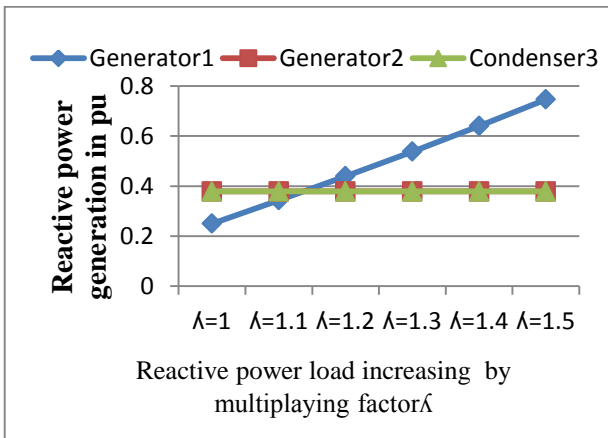


Fig. 8: Reactive power generation by the Generator and thermal condenser

Fig. 9: Reactive power generation by the Generators and thermal and hydro condenser

Table 2. Reactive power generation in different cases with increasing reactive power load in MVar

| Cases | Reactive sources | MVar at $\lambda=1.0$ | \$/hr at $\lambda=1.1$ | MVar at $\lambda=1.1$ | \$/hr at $\lambda=1.2$ | MVar at $\lambda=1.2$ | \$/hr at $\lambda=1.3$ | MVar at $\lambda=1.3$ | MVar at $\lambda=1.4$ | \$/hr at $\lambda=1.4$ | MVar at $\lambda=1.5$ | \$/hr at $\lambda=1.5$ |
|---------|------------------|-----------------------|------------------------|-----------------------|------------------------|-----------------------|------------------------|-----------------------|-----------------------|------------------------|-----------------------|------------------------|
| Case 1a | G1 | 25.06 | 0 | 19.66 | 0 | 23.8 | 0 | 28.02 | 32.34 | 0 | 36.75 | 0 |
| | G2 | 37.86 | 201.913 | 37.86 | 201.913 | 37.86 | 201.913 | 37.86 | 37.86 | 201.913 | 37.86 | 201.913 |
| | hydro condenser | 37.86 | 407.0822 | 51.12 | 432.3391 | 55.83 | 457.9177 | 60.6 | 65.45 | 483.9254 | 70.37 | 510.3084 |
| Case 1b | G1 | 25.06 | 0 | 26.27 | 0 | 29.38 | 0 | 32.5 | 35.65 | 0 | 38.81 | 0 |
| | G2 | 37.86 | 201.913 | 37.86 | 201.913 | 37.86 | 201.913 | 37.86 | 37.86 | 201.913 | 37.86 | 201.913 |
| | condenser | 37.86 | 574.5818 | 37.86 | 574.5818 | 37.86 | 574.5818 | 37.86 | 37.86 | 574.5818 | 37.86 | 574.5818 |
| Case 1c | hydro condenser | -- | 164.9162 | 5.96 | 189.476 | 10.54 | 214.0894 | 15.13 | 19.74 | 238.81 | 24.38 | 263.6916 |
| | G1 | 25.06 | 0 | -5.16 | 0 | -3.68 | 0 | -2.17 | -0.62 | 0 | 3.59 | 0 |
| | hydro condenser | 37.86 | 522.1593 | 72.58 | 561.1976 | 79.86 | 600.8793 | 87.26 | 94.79 | 641.2582 | 100 | 669.1963 |
| Case 2a | condenser | 37.86 | 600.8743 | 40 | 600.8743 | 40 | 600.8743 | 40 | 40 | 600.8743 | 40 | 600.8743 |
| | G1 | 25.06 | 0 | 21.24 | 0 | 25.89 | 0 | 30.58 | 35.31 | 0 | 40.09 | 0 |
| | G2 | 37.86 | 201.913 | 37.86 | 201.913 | 37.86 | 201.913 | 37.86 | 37.86 | 201.913 | 37.86 | 201.913 |
| | condenser | 37.86 | 574.5818 | 37.86 | 574.5818 | 37.86 | 574.5818 | 37.86 | 37.86 | 574.5818 | 37.86 | 574.5818 |
| Case 2b | Hydro condenser | -- | 184.757 | 9.66 | 202.9356 | 13.05 | 221.275 | 16.47 | 19.93 | 239.8289 | 23.42 | 258.5437 |
| | G1 | 25.06 | 0 | 24.72 | 0 | 28.96 | 0 | 33.24 | 37.56 | 0 | 41.92 | 0 |
| | G2 | 37.86 | 201.913 | 37.86 | 201.913 | 37.86 | 201.913 | 37.86 | 37.86 | 201.913 | 37.86 | 201.913 |
| | condenser | 37.86 | 574.5818 | 37.86 | 574.5818 | 37.86 | 574.5818 | 37.86 | 37.86 | 574.5818 | 37.86 | 574.5818 |
| Case 2c | Hydro condenser | -- | 170.922 | 7.08 | 190.8166 | 10.79 | 210.9256 | 14.54 | 18.32 | 231.1954 | 22.14 | 251.6798 |
| | G1 | 25.06 | 0 | 24.87 | 0 | 28.61 | 0 | 32.39 | 36.19 | 0 | 40.02 | 0 |
| | G2 | 37.86 | 201.913 | 37.86 | 201.913 | 37.86 | 201.913 | 37.86 | 37.86 | 201.913 | 37.86 | 201.913 |
| | condenser | 37.86 | 574.5818 | 37.86 | 574.5818 | 37.86 | 574.5818 | 37.86 | 37.86 | 574.5818 | 37.86 | 574.5818 |
| Case 2c | Hydro | -- | 170.3322 | 6.97 | 192.2108 | 11.05 | 214.1966 | 15.15 | 19.28 | 236.3433 | 23.43 | 258.5973 |

4. Conclusions

In this work, reactive power cost analysis has been carried out for a synchronous generator and the condensers. Cost characteristic curves have been obtained for synchronous generator and cost of condensers has also been obtained. These cost characteristics have been considered for obtaining the reactive power cost in IEEE 14 bus test system without and with condensers. The condensers have been placed at respective buses where voltage profile is low. The reactive power requirements have been obtained with increased load requirements in a system for different cases and the reactive support have been obtained from the synchronous generators and the condensers at each increased load demand. With support of condensers, the reactive cost has been obtained for all the cases. It is observed that with the support of hydro condenser with increasing demand, the reactive power requirement from the generators is lower. The condensers can be remunerated based on the cost calculated for their reactive support. The voltage profile is better with reactive support and losses also reduce in the system. The analysis for the cost remuneration for reactive support can help the system operator to procure reactive power and accordingly collect the price from the market participants and can pay the price for the ancillary services providers.

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