

Hydraulic Model of Trickle Irrigation Laterals with Single and Varying Pipe Size

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Abstract. An important objective of any trickle system is a uniform distribution of water delivered through the emitters. Computation of flow distribution requires knowledge of the variables such as pressure, flow rate, length of lateral, characteristics of the orifices, and frictional loss in the system. The problem of design of trickle laterals pipe with equally spaced emitters, uniform slope and single or combination of diameters of different lengths is investigated. The friction head loss along the lateral is estimated by the Darcy-Weisbach formula, taking into account the variation of the Reynolds number, and the friction coefficient formula corresponding to each zone. In addition, the net pressure head along the lateral with single or varying size is evaluated.

A software computer program is presented for the hydraulic analysis and designing trickle irrigation laterals, very simple and yet powerful design curves for different flow regimes are presented.

Keywords: Trickle irrigation; Friction losses; Pipe flow; Hydraulic design

1. Introduction

The pressure head between $X=0$ and $X=x$ along the lateral by the following equation:

$$h_0 + \frac{Q_0^2}{2gA^2} + z_0 = h_x + \frac{Q_x^2}{2gA^2} + z_x + hf_{(0,x)} \quad (1)$$

In which A = the cross-sectional area of the lateral; z_0 and z_x = elevations of lateral at $X=0$ and $X=x$, respectively, above an arbitrary datum; g = the acceleration due to gravity; Q_0^2 and Q_x^2 = the lateral discharge at $X=0$ and $X=x$; $hf_{(0,x)}$ = the friction head loss between $X=0$ and $X=x$. Solving Eq.(1) for $h_0 - h_x$, we have:

$$h_0 - h_x = (z_x - z_0) + \frac{1}{2gA^2} (Q_x^2 - Q_0^2) + hf_{(0,x)} \quad (2)$$

If the lateral pipe has a uniform slope S_0 , the difference in levels of points 0 and x may be given by

$$(z_x - z_0) = S_0 x \quad (3)$$

The negative sign for laterals sloping downward and positive sign for upward slopes.

The lateral discharge between the $X=0$ and $X=x$ can be evaluated from:

$$Q_x = Q_0 - n_x q \quad (4)$$

$$Q_0 = \frac{L}{S} q \quad (5)$$

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$$n_x = \frac{x}{S} \quad (6)$$

n_x the number of emitter at $X=x$; q emitter discharge (m^3/sec), L the lateral length (m), S the emitter space (m).

Substituting from (5) and (6) into (4) and simplifying

$$Q_x = Q_0 - \frac{x}{s} \frac{S}{L} Q_0 \quad (7)$$

$$Q_x = Q_0 \left(1 - \frac{x}{L}\right) \quad (8)$$

$$\frac{1}{2gA^2} (Q_x^2 - Q_0^2) = -\frac{1}{2gA^2} Q_0^2 \left[1 - \left(1 - \frac{x}{L}\right)^2\right] \quad (9)$$

The most commonly used equation to compute this head loss is the Darcy-Wiesbach equations as given by

$$hf = f \frac{V^2}{2gD} \quad (10)$$

The Darcy-Weisbach formula is the more accurate and widely used formula for most engineering applications. The friction factor h_f is determined as a function of the Reynolds number and the relative roughness of the pipe. For drip irrigation systems, the smooth pipe relationships in the Moody diagram can be used to estimate f [1]. The value of the friction factor f is determined as follows:

$$f = \frac{64}{R} \quad \text{for} \quad R \leq 2000 \quad (11)$$

Though not used in design, the friction factor in the transition zone $2000 < R < 3000$ can be approximated by a constant value consistent with Watters and Keller [1] results for most pipe sizes

$$f = 0.04 \quad \text{for} \quad 2000 < R \leq 3000 \quad (12)$$

The Blasius equation can be used to calculate the friction factor f as

$$f = \frac{0.32}{R^{0.25}} \quad \text{for} \quad 3000 < R \leq 10^5 \quad (13)$$

And from Watters and Keller [1]

$$f = \frac{0.13}{R^{0.172}} \quad \text{for} \quad 10^5 < R \leq 10^7 \quad (14)$$

The Reynolds number R can be expressed as a function of Q and D as

$$R = \frac{VD}{\nu} = \frac{4Q}{\pi\nu D} = 1.27 \times 10^6 \frac{Q}{D} \quad (15)$$

With the kinematic viscosity value of $1 \times 10^{-6} m^2/s$ for an average water temperature. Q in m^3/s and in D m in Eq. (15)

If the value of R is substituted into Eqs.(11)-(14) and the corresponding values of f into Eq.(10), a general expression for friction head loss along lateral with decreasing flow can be calculated by

$$hf = k_i \frac{Q_i^{p_i-3}}{D^{p_i}} \quad (16)$$

Table 1: Values of the Variables for Eq. 8

| i | k_i | p_i | R |
|---|-------------------------|-------|----------------------|
| 1 | 4.1969×10^{-6} | 4.00 | $R < 2000$ |
| 2 | 3.3051×10^{-3} | 5.00 | $2000 < R < 3000$ |
| 3 | 7.8918×10^{-4} | 4.75 | $3000 < R \leq 10^5$ |
| 4 | 9.5896×10^{-4} | 4.828 | $10^5 < R \leq 10^7$ |

Table 1 gives the values of k_i and P_i , according to the friction factor equation and the units used in the equations, i.e., Q in m^3/s , D in m , and L in m .

Integration of Eq. (16) with the use of Eq.(8) results in

$$hf_x = k_i \frac{Q_0^{P_i-3}}{(P_i-2)D^{P_i}} L \left(1 - \frac{x}{L}\right)^{P_i-2} + C \quad (17)$$

The integration constant C is the value of H at the distal end of the lateral ($X=L$). The solution results in a drop in total head from the beginning ($X=0$) to any position x of:

$$hf_{(0,x)} = k_i \frac{Q_0^{P_i-3}}{(P_i-2)D^{P_i}} \alpha.L \left[1 - \left(1 - \frac{x}{L}\right)^{P_i-2}\right] \quad (18)$$

In which α is an equivalent barb coefficient, Barb coefficient was computed for emitter connections according to Pitts et al., and Amer and Gomaa [2]-[3] as follows:

$$\alpha = 1 + \frac{0.01d}{SD^{1.9}} \quad (19)$$

Where, α is an equivalent barb coefficient, d is outer diameter of emitter barb in m , D is the inner pipe diameter in m , S is emitter spacing in m .

And a total drop in pressure of

$$hf_L = k_i \frac{Q_0^{P_i-3}}{(P_i-2)D^{P_i}} \alpha.L \quad (20)$$

Substituting from (3), (9), and (18) into (2) and simplifying

$$h_{(o,x)} = S_0 x - \frac{Q_0^2}{2gA^2} \left[1 - \left(1 - \frac{x}{L}\right)^2\right] + k_i \frac{Q_0^{P_i-3}}{(P_i-2)D^{P_i}} \alpha.L \left[1 - \left(1 - \frac{x}{L}\right)^{P_i-2}\right] \quad (21)$$

Then the total change in pressure at the end of lateral $x = L$ (m).

$$h_{(o,L)} = S_0 L - \frac{Q_0^2}{2gA^2} + k_i \frac{Q_0^{P_i-3}}{(P_i-2)D^{P_i}} \alpha.L \quad (22)$$

2. Design Laterals with Single or Varying Pipe Size

Assume for designing laterals diameters the velocity head neglected then the Eq.(22)write as following:

$$h_{(o,L)} = S_0 L + k_i \frac{Q_0^{P_i-3}}{(P_i-2)D^{P_i}} \alpha.L \quad (23)$$

According to ASAE [4], pressure difference throughout the system or block or sub-unit should be maintained within such a range that the design emission uniformity (EU) is obtained.

In order to achieve this, Keller and Bliesner [5] propose the following equation:

$$\Delta H_s = 2.5 \times [H_a - H_m] \quad (24)$$

$$EU = 100 \times (1.0 - 1.27 C_v / \sqrt{N_p}) \times (q_m - q_a) \quad (25)$$

$$T_a = \frac{IR_g}{N_p \times q} \quad (26)$$

Where T_a duration of irrigation per day (hr), IR_g gross irrigation requirement (mm/day), EU the design emission uniformity(%), N_p number of emitter per plant, C_v the manufacture's coefficient of variation, q_m the minimum emitter discharge for minimum emitter in the sub-unit (lph), q_a the average or design emitter discharge for the sub-unit(lph), ΔH_s = allowable pressure variation that will give an EU reasonable close desired design value (m), H_a pressure head that will give the q_a required to satisfy Eq. (26) with the EU required in Eq. (25), H_m pressure head that will give the required q_m to satisfy EU in Eq. (25)

Unfortunately, only serious manufacturers provide data on emitter manufacture's variability and emitter exponents. Engineers are therefore obliged to design without this information. In such case the allowable

pressure variation is estimated to 10% of the emitter operation pressure. Azenkot [6] Suggest that for emitter with exponent $n=0.5$ the allowable pressure variability should be 20% of the operation pressure, limiting thus the flow variability among emitter to 10%.

A properly designed lateral will operate within the design constraints as long as the actual head loss due to friction along the lateral is less than the allowable head loss given by the above equation Eq. (23) [7].

The head loss along a two diameter lateral as shown in Fig. 1 can be computed by Hart, Keller and Bliesner [5]-[8]

$$h_{(o,L)} = hf_{D1(0,L)} - hf_{D1(L1,L2)} + hf_{D2(L1,L2)} \quad (27)$$

Where $h_{(o,L)}$ = actual head loss in the lateral m; $hf_{D1(0,L)}$, $hf_{D1(L1,L2)}$ and $hf_{D2(L1,L2)}$ = given in terms of D_2, L_2, Q_2, D_1, L and Q_0 .

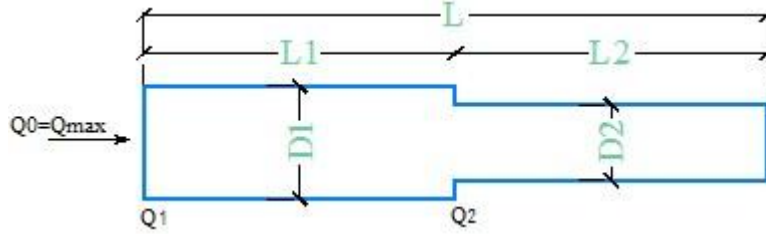


Fig. 1: flow through laterals with pipes of two diameters

The head losses $hf_{D1(0,L)}$, $hf_{D1(L1,L2)}$ and $hf_{D2(L1,L2)}$ can be computed using Eq.(22) as

$$h_{D1(0,L)} = S_0 L + k_i \frac{Q_0^{p_i-3}}{(p_i-2)D_1^{p_i}} \alpha_1 L \quad (28)$$

$$h_{D1(L1,L2)} = S_0 L_2 + k_i \frac{Q_2^{p_i-3}}{(p_i-2)D_1^{p_i}} \alpha_1 L_2 \quad (29)$$

$$h_{D2(L1,L2)} = S_0 L_2 + k_i \frac{Q_2^{p_i-3}}{(p_i-2)D_2^{p_i}} \alpha_2 L_2 \quad (30)$$

The flow rate at the entrance of each section can be computed as a function of the drip emitter outlet flow rate and number of emitters. Then

$$Q_0 = \frac{L}{S} q \quad (31)$$

$$Q_2 = \frac{L_2}{S} q \quad (32)$$

Substituting Eq.(30) and (31) into Eqs.(27)–(29) and then to Eq.(26), we get

$$h_{(o,L)} = S_0 L + k_i \frac{\left(\frac{L}{S} q\right)^{p_i-3}}{(p_i-2)D_1^{p_i}} \alpha_1 L - S_0 L_2 - k_i \frac{\left(\frac{L_2}{S} q\right)^{p_i-3}}{(p_i-2)D_1^{p_i}} \alpha_1 L_2 + S_0 L_2 + k_i \frac{\left(\frac{L_2}{S} q\right)^{p_i-3}}{(p_i-2)D_2^{p_i}} \alpha_2 L_2 \quad (33)$$

$$\frac{(h_{(o,L)} - S_0 L) \times (p_i - 2) S^{p_i-3}}{k_i q^{p_i-3}} - \frac{L^{p_i-2}}{D_1^{p_i}} \alpha_1 = -\frac{L_2^{p_i-2}}{D_1^{p_i}} \alpha_1 + \frac{L_2^{p_i-2}}{D_2^{p_i}} \alpha_2 \quad (34)$$

$$\frac{(h_{(o,L)} - S_0 L) \times (p_i - 2) S^{p_i-3}}{k_i q^{p_i-3}} - \frac{L^{p_i-2}}{D_1^{p_i}} \alpha_1 = L_2^{p_i-2} \left(\frac{\alpha_2}{D_2^{p_i}} - \frac{\alpha_1}{D_1^{p_i}} \right) \quad (35)$$

Solving for the length of the second pipe, we get

$$L_2 = \left[\left(\frac{(h_{(o,L)} - S_0 L) \times (p_i - 2) S^{p_i-3}}{k_i q^{p_i-3}} - \frac{L^{p_i-2}}{D_1^{p_i}} \alpha_1 \right) \times \left(\frac{\alpha_2}{D_2^{p_i}} - \frac{\alpha_1}{D_1^{p_i}} \right)^{-1} \right]^{\frac{1}{p_i-2}} \quad (36)$$

The third diameter, if needed, will be determined using the length of the second diameter calculated and with the head loss as the difference between the original allowable head loss and that attributed for the larger diameter in the first step.

The total head loss along a three diameter lateral as shown in Fig. 2 can be computed as follows:

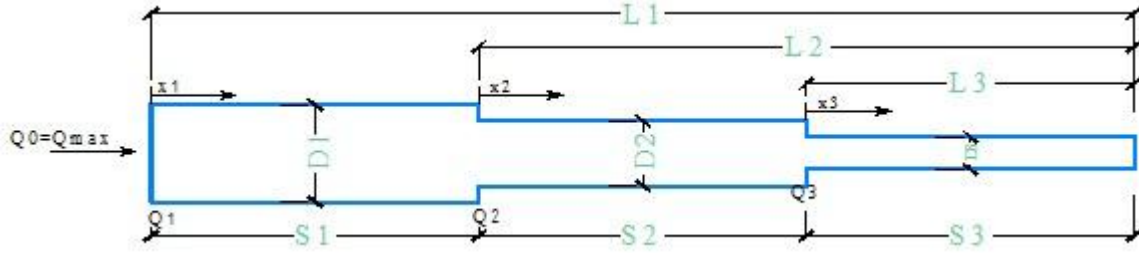


Fig. 2: Three different sizes of three lateral sections sketch.

$$h_{(s_1)} = S_0 x_1 - \frac{Q_1^2}{2gA_1^2} \left[1 - \left(1 - \frac{x_1}{L_1} \right)^2 \right] + k_i \frac{Q_1^{p_i-3}}{(p_i-2)D_1^{p_i}} \alpha_1 \cdot L_1 \cdot \left[1 - \left(1 - \frac{x_1}{L_1} \right)^{p_i-2} \right] \quad (37)$$

Where $h_{(s_1)}$ is the total change in head at first section (m).

$$h_{(s_2)} = S_0 x_2 - \frac{Q_2^2}{2gA_2^2} \left[1 - \left(1 - \frac{x_2}{L_2} \right)^2 \right] + k_i \frac{Q_2^{p_i-3}}{(p_i-2)D_2^{p_i}} \alpha_2 \cdot L_2 \cdot \left[1 - \left(1 - \frac{x_2}{L_2} \right)^{p_i-2} \right] \quad (38)$$

Where, $h_{(s_2)}$ is the total change in head at second section (m).

$$h_{(s_3)} = S_0 x_3 - \frac{Q_3^2}{2gA_3^2} \left[1 - \left(1 - \frac{x_3}{L_3} \right)^2 \right] + k_i \frac{Q_3^{p_i-3}}{(p_i-2)D_3^{p_i}} \alpha_3 \cdot L_3 \cdot \left[1 - \left(1 - \frac{x_3}{L_3} \right)^{p_i-2} \right] \quad (39)$$

Where, $h_{(s_3)}$ is the total change in head at the third section (m).

$$h_{(o,L)} = h_{(s_1)} + h_{(s_2)} + h_{(s_3)} \quad (40)$$

Where, $h_{(o,L)}$ is the total change in pressure at the total section (m).

Hydraulic power loss along lateral was determined as follows:

$$P_{HL} = h_{(o,L)} \cdot Q \cdot \gamma \quad (41)$$

Where, P_{HL} is hydraulic power loss in watt, and Q is lateral inlet discharge in m^3/s , and γ water specific weight in N/m^3 .

3. Software Description

The methodology described here was implemented using for a computer program using the Microsoft Visual Basic 6.0.

The Design of Trickle Irrigation Laterals program has three options:

- 1- The Design of Trickle Irrigation Laterals program allows the computation of lateral diameter and head distribution along a lateral of constant slope. Data required for the computation are the spacing of outlet (m), length of the lateral(m), allowable head loss (m), slope of the lateral (negative sign is for laterals sloping downward and the positive for upward slopes), average emitter discharge(m^3/sec).
- 2- If we were to design the system using the two available diameters, the design involves determining the length of the two sizes that will satisfy the allowable head loss requirement. The program allows the computation of lengths of the two sizes for corresponding two diameters and head distribution along lateral with two diameters of constant slope. Data required for the computation are the same last data in additional to the two diameters.
- 3- If we were to design the system using the three available diameters, the length also needs to change accordingly by subtracting the length of the largest diameter calculated from the original total length. The new length for the second and third diameters and the new head loss are used to size the two smaller diameters. The program also allows the computation of lengths of the three sizes for

corresponding three diameters and head distribution along laterals with three sizes of constant slope. Data required for the computation are the same last data in addition to the three diameters (mm).

Executing the program will show a display as in Fig 3 (a). Once these data are entered, the length of the laterals, head losses and head distribution are calculated and displayed as shown in Fig 3 (b), Fig 3 (c) and Fig 3 (d).

SALAHOU-Design of Trickle Irrigation Laterals

Length of pipe (m)

Spacing of outlet (m)

emitter flow rate m³/s

allowable head loss

Uniform Slope S₀

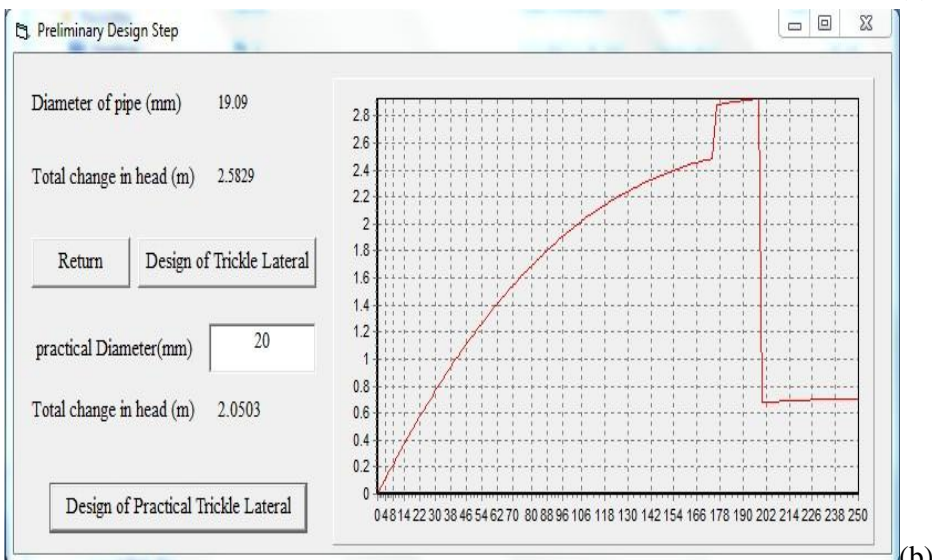
Design Options

Design lateral's Diameter and head

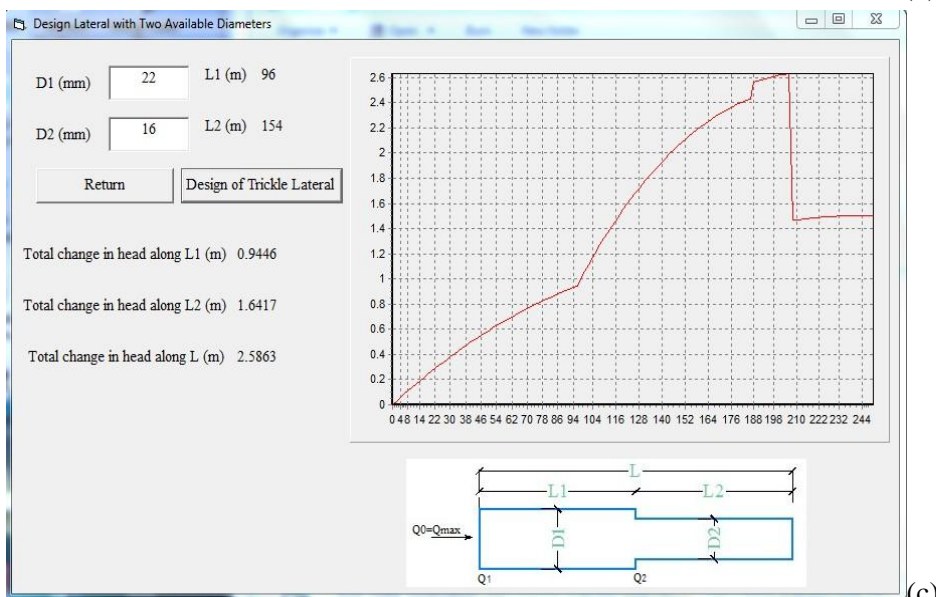
Design Lateral with Two Available Diameters

Design Lateral with Three Available Diameters

(a)



(b)



(c)

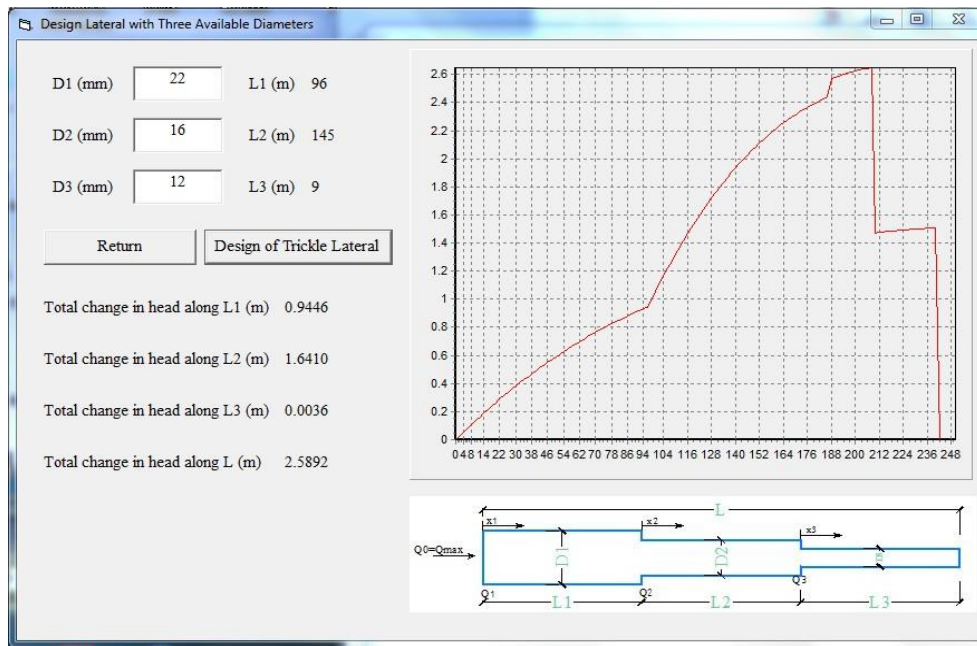


Fig.3: Irrigation Lateral Computations, Data are entered as shown in (a). The head distribution along the lateral and lateral lengths shown in (b, c, d)

Notice: the curve shows that the head change exceed 2.6m (allowable pressure variation) because the flow condition is changed ($2000 < R < 3000$) and the values above 2.6m is not used in design.

4. Example

4.1. Given

A 250 m drip lateral with spacing of emitters 2 m apart and an emitter flow rate of $1.2 \times 10^{-6} \text{ m}^3/\text{sec}$ is to be the design for an allowable head loss of 2.6 m. Design the system for the given head loss.

4.2. Solution

We have Eq. (23) as

$$h_{(o,L)} = S_0 L + k_i \frac{Q_0^{p_i-3}}{(p_i - 2) D^{p_i}} \alpha L$$

In this case, we have

$$S_0=0, p_i = 4.75, k_i = 7.8918 \times 10^{-4}, h_{(o,L)} = \Delta H_S = 2.6 \text{ m}, Q_0 = 1.2 \times 10^{-6} \frac{250}{2} = 1.5 \times 10^{-4} \text{ m}^3 / \text{s Subs}$$

tituting these values into Eq.(23), we get a diameter of 0.01909m = 19.09mm if we were to design for one diameter only. Substituting these values into Eq.(22), we get a head of 2.6m (the same of allowable pressure variation).

If the 20 mm lateral is used, its friction loss would be 2.05 m

Assume there are two diameters available, i.e., 22 and 16 mm

Thus, by using Eq. (36) for $h_{(o,L)} = \Delta H_S = 2.6 \text{ m}$, $S_p=2\text{m}$, $k_i = 7.8918 \times 10^{-4}$, $P= 4.75$, and $q= 1.2 \times 10^{-6} \text{ m}^3/\text{s}$, we get $L_2 = 154 \text{ m}$. The lateral design requires the use of 154 m of the 16 mm and 96 m of the larger diameter 22 mm.

Substituting these values into Eq. (22), we get a head along the lateral which has the first diameter of 0.944m and a head along the lateral which has the second diameter of 1.642m and the total head in the lateral which has two diameters is 2.6m(equal to allowable pressure variation).

Assume there are three diameters available, i.e., 22, 16 and 12mm, if we were to design the system using the three available diameters.

$D_1=22 \text{ mm}$, $S_1=96\text{m}$, $h_{(S1)} = 0.944\text{m}$, the new allowable head loss for the 154 m length is 1.656 m

Using Eq. (35) with $h_{(0,L)} = 1.656$ m, $L_2 = 154$ m, $D_2 = 16$ mm, $D_3 = 12$ mm, $Sp = 2.0$ m, $P = 4.75$, and $q = 1.2 \times 10^{-6} \text{ m}^3/\text{s}$, the length of the smallest diameter pipe is determined to be 9 m. The next larger diameter 16 mm will be 145 m long. The design will have three pipe sizes 22, 16, and 12 mm with the corresponding lengths of 96, 145, and 9 m, respectively.

Substituting these values into Eq. (22), we get a head along the lateral which has the first diameter of 0.944m, the head along the lateral which has the second diameter of 1.64m, the head along the lateral which has the third diameter of 0.004m and the total head along the lateral which has three diameters is 2.6m.

5. Conclusion

Optimum design of trickle irrigation lateral could be done by managing lateral length, size, emitter discharge, inlet pressure, and emitters spacing with selecting high quality of emitters. Optimum lateral dimension should be constrained by friction loss, flow variation or emission uniformity of trickle unit. For that purpose, an analytical solution that counts for the pressure head change and the variation of the Reynolds number along a trickle irrigation laterals with single and varying pipe size is presented.

The Design of Trickle Irrigation Laterals computer program give an opportunity for more accurate hydraulic design of trickle laterals. The model is theoretically based and the program is open for substituting with different lateral parameters, the program allows the computation of lengths of the varying pipe sizes for corresponding allowable pressure variation, head distribution along laterals and developing design curves, which gives economical solution for the initial cost (cost of laterals) and operational cost.

6. Acknowledgments

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7. References

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Appendix A.

The following symbols are used in this technical note:

| | |
|---|---|
| D_1 = diameter of the first pipe; | D_2 = diameter of the second pipe; |
| α = an equivalent barb coefficient; | d = outer diameter of emitter barb in (m); |
| D = the inner pipe diameter in(m); | S = emitter spacing in (m); |
| Q_0 = the lateral discharge at $X=0$ position (m^3/sec); | Q_x = the lateral discharge at any position x (m^3/sec); |
| f = Darcy-Weisbach friction factor; | g = gravitational acceleration, $9.81 \text{ m}/\text{sec}^2$; |
| L = length of pipe m; | Q_1 = flow rate in the entire first section; |
| Q_2 = flow rate in the entire second section; | Q_3 = flow rate in the entire third section; |
| q = emitter flow rate (m^3/sec); | S_o = Uniform slope; |
| R = Reynolds number (dimensionless); | S_p = spacing of outlets on a lateral (m); |

V = average velocity of flow in the pipe (m/s);
 IR_g = gross irrigation requirement (mm/day);
 N_p = number of emitter per plant;
 q_m = the minimum emitter discharge for minimum emitter in the sub-unit (lph);
 ΔH_s = allowable pressure variation (m);

 H_m = pressure head that will give the required q_m to satisfy EU in Eq.(24) (m);
 $h_{(s_2)}$ = the total change in head at second section (m);
 $h_{(o,L)}$ = the total change in head at the total section (m);
 γ = water specific weight in N/m^3 .

T_a = duration of irrigation per day (hr);
 EU = the design emission uniformity (%);
 C_v = the manufacture's coefficient of variation;
 q_a = the average or design emitter discharge for the sub-unit(lph);
 H_a = pressure head that will give the q_a required to satisfy Eq.(25) with the EU required in Eq.(24) (m);
 $h_{(s_1)}$ = the total change in head at first section (m);
 $h_{(s_3)}$ = the total change in head at the third section (m);
 P_{HL} is hydraulic power loss in watt; and