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Trading Pollution Discharge Permits in Rivers Using Fuzzy Bi-matrix Games

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***Abstract*—This paper presents a new game theoretic methodology for trading pollutant discharge permits in rivers by utilizing bi-matrix games with fuzzy goals, Non-dominated Sorting Genetic Algorithms II (NSGA-II) and cooperative game theory. In this methodology, a trade-off curve between objectives, namely average treatment level of dischargers and fuzzy risk of low water quality is obtained using the NSGA-II. Then, the best non-dominated solution is selected using a non zero-sum bi-matrix game with fuzzy goals. In the next step, by forming some possible coalitions among dischargers, treatment costs are reallocated among dischargers participating in a coalition and side payments are calculated. The applicability and efficiency of the methodology are examined in a real-world case study.**

Keywords-Trading Pollutant Discharge Permits; Fuzzy Bi-matrix Game; Water Qualiy

I. Introduction

In Tradable Discharge Permit (TDP) programs, the action to pollute is defined as a property right with tangible value which is transferable. In TDP programs, dischargers who can cost-efficiently remove pollutants will remove more than required, selling unused permits to relatively inefficient dischargers, who will be buyers of pollutant discharge permits, discharging more wastewater than permitted by their initial pollutant discharge rights [1]. In the past decades, many studies have been carried out for developing deterministic and stochastic TDP programs (e.g. [2, 3, 4, 5, 6 and 7]).

In this Paper, for the first time, bi-matrix games with fuzzy goals are utilized for developing efficient strategies for trading pollutant discharge permits in rivers.

II. Model Framework

In the first step of the proposed methodology, the NSGA-II [8] provides a trade-off between the objectives, namely the average treatment level of dischargers and a fuzzy risk of low water quality. Then, the best non-dominated solution on the trade-off curve is selected using fuzzy bi-matrix game.

For fairly reallocation of treatment cost to participants in a coalition and determining side payments, a single objective optimization model and cooperative game theory are utilized. The fuzzy risk at each checkpoint along the river can be estimate as follows [9]:

$$FR = \sum_{c_{\min}}^{c_{\max}} \mu_W(c)\, p(c) \tag{1}$$

where, $\mu_W(c)$ is the membership function of the fuzzy event of low water quality based on the concentration of a water quality indicator (c), $c_{\min}$ and $c_{\max}$ are respectively minimum and maximum concentration level of the water quality indicator and $p(c_l)$ is probability density function (PDF) of the concentration level of the water quality indicator.

In this paper, PDF of the concentration of water quality indicator at checkpoints along the river are obtained using Monte Carlo simulations considering the PDF of random variables in the water quality simulation model, which has been linked with the optimization model.

To select the best non-dominated solution from the trade-off of the conflicting objectives of environmental protection agencies and dischargers, a fuzzy bi-matrix game is utilized. A bi-matrix game with fuzzy goal is shown to be equivalent to a crisp non-linear programming problem in which the objective as well as all constraint functions are linear except two constraint functions, which are quadratic [10].

Let R^n denote the n-dimensional Euclidean space and R^n be its non-negative. Let $A, B \in R^{m\times n}$ be $(m\times n)$ real matrices and $e^T = (1,1,....,1)$ be a vector of 'ones' whose dimension is specified as per the specific context.

A crisp bi-matrix game (BG) can be shown as:

$$BG = (S^m, S^n, A, B) \tag{2}$$

where:

$$S^m = \{x \in R^m, e^T x = 1\} \tag{3}$$

$$S^n = \{y \in R^n, e^T y = 1\} \tag{4}$$

S^m and S^n are called the strategy space of Players I and II, respectively. A and B are also called the pay-off matrices of Players I and II, respectively. Now let v_0 and w_0 be scalars representing the aspiration levels of Players I and II, respectively. Then a bi-matrix game with fuzzy goals, denoted by BGFG, is defined as [10]:

$$\text{BGFG: } (S^m, S^n, A, B, v_0, \tilde{\gtrsim}, w_0, \tilde{\lesssim}) \tag{5}$$

where, '$\tilde{\lesssim}$' and '$\tilde{\gtrsim}$' are the fuzzified versions of '$\le$' and '$\ge$', respectively. By defining tolerances p_0 and p_0' for Player I, and, q_0 and q_0' for Player II, BGFG is shown as:

$$\text{BGFG: } (S^m, S^n, A, B, v_0, p_0, p_0'; w_0, q_0, q_0'; \tilde{\gtrsim}, \tilde{\lesssim}) \quad (6)$$

Employing the Zimmermann's approach, the crisp equivalent of the fuzzy non-linear programming is obtained [11]:

$$(NLP): \max_{x\in S^m, y\in S^n} \{\mu_1(A_1 y),\ldots,\mu_m(A_m y), \nu_1(x^T B_1),\ldots,\nu_n(x^T B_n), \mu_0(x^T A_y), \nu_0(x^T B_y)\} \quad (7)$$

or:

$$\max \ \lambda \quad (8)$$

Subject to:

$$\lambda \le 1 - \frac{A_{iy} - v_0}{p_0} \quad (i = 1,2,\ldots,m) \quad (9)$$

$$\lambda \le 1 - \frac{x^T B_j - w_0}{q_0} \quad (j = 1,2,\ldots,n) \quad (10)$$

$$\lambda \le 1 + \frac{x^T A_y - v_0}{p_0'} \quad (11)$$

$$\lambda \le 1 + \frac{x^T B_y - w_0}{q_0'} \quad (12)$$

$$x \in S^m, \ y \in S^n, \ \lambda \in [0,1] \quad (13)$$

BGFG provides a non-dominated solution which shows the initial treatment levels of dischargers and corresponding fuzzy risk of having low water quality along the river.

In a cooperative trading pollutant discharge permits trading, dischargers can form coalitions to reduce their treatment costs. Cooperative cost allocation is attractive, but a key issue, fairness, still needs to be dealt with carefully in order to have cooperation. By minimizing the total or average treatment cost, the fairness criterion is not satisfied. Niksokhan et al. (2009) utilized cooperative game theory for fairly treatment cost reallocation [7]. By comparing the initial and reallocated treatment costs of dischargers, the values of side payments among dischargers are defied and the buyer and sellers of discharge permits are determined. In this paper, the proposed cost reallocation methodology is proposed by Niksokhan et al. (2009) is used [7]. In that methodology, the total treatment cost is reallocated through two sub-models: Maximization of Total Treatment Cost Reduction (MTTCR) and the Cooperative Reallocation Game (CRG).

III. Results and Discussion

To evaluate the efficiency and applicability of the proposed trading discharge permit methodology, it is applied to the Zarjub river. The main characteristics of this river and its pollution loads can be found in [6 and 12]. In this paper, the fuzzy risk of low water quality at checkpoints is calculated using a hypothetic fuzzy membership function for the DO concentration (Fig. 1).

To calculate the fuzzy risk, the Monte Carlo analysis is utilized considering the main random variables in the water quality simulation model, namely upstream river flow, headwater temperature, BOD concentration in headwater, BOD and DO concentrations in discharging wastewaters, the BOD decay rate (k) and the reaeration coefficient (k_2). The river water quality is simulated using Streeter-Phelps equations [13]. Five hundred Monte Carle simulations are done to calculate the PDF of DO at several checkpoints along the river. The NSGA-II provides the trade-off between the average treatment level of dischargers and the fuzzy risk (Fig. 2).

In the BGFG, the pay-off matrices considered for dischargers (matrix A) and environmental protection agencies (matrix B) are as follows:

$$\begin{array}{r} \\ 10 \\ 30 \\ A = 50 \\ 70 \\ 90 \end{array}\begin{array}{c} \begin{array}{ccccc} 42 & 46 & 50 & 54 & 58 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0.85 \\ 0 & 0 & 0 & 0.62 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0.39 & 0 & 0 & 0 \\ 0.31 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \qquad \begin{array}{r} \\ 10 \\ 30 \\ B = 50 \\ 70 \\ 90 \end{array}\begin{array}{c} \begin{array}{ccccc} 42 & 46 & 50 & 54 & 58 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0.15 \\ 0 & 0 & 0 & 0.38 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0.61 & 0 & 0 & 0 \\ 0.69 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

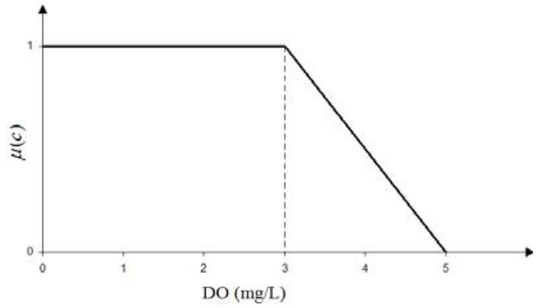

Figure 1. The selected fuzzy membership function for low water quality

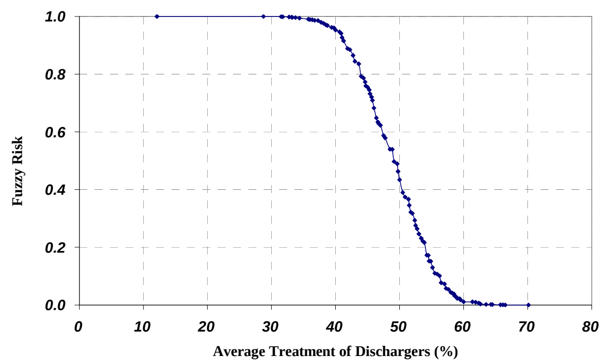

Figure 2. Trade-off between objectives obtained using the NSGA-II

Vectors of (10, 30, 50, 70, 90) and (42, 46, 50, 54, 58) are the strategy spaces of dischargers and environmental protection agencies. The pay-off matrices have been provided by some experts from environmental protection agencies and dischargers considering the trade-off curve between objectives. In this paper, the parameters of the BGFG are assumed to be:

$v_0 = 0.2$, $w_0 = 0.4$, $p_0 = 0.1$, $q_0 = 0.2$, $p'_0 = 0.2$ and $q'_0 = 0.4$.

The optimum value for the objective function of this optimization model is $\lambda^* = 0.599$. Tables 1 and 2 presents the characteristics of the selected non-dominated solution using BGFG. This solution is selected by calculating the expected values of the strategies of both dischargers and environmental protection agencies. The value of $\lambda^* = 0.599 < 1$, indicates that it is a fuzzy scenario and then it is the least degree up to which the goals of both the players are satisfied.

Comparing the treatment costs of dischargers shows that dischargers 2, 6 and 7 can participate in a coalition to reduce the total treatment cost. Dischargers 1 and 3 have no treatment cost and dischargers 4 and 5 should completely treat their pollution loads because they have lower treatment costs and increasing their treatment levels does not significantly improve the river water quality. For this coalition, the maximum cost reduction and treatment cost of dischargers 2, 6 and 7 are calculated (Table 3).

TABLE I. OPTIMAL VALUES FOR THE VARIABLES IN THE NON-LINEAR OPTIMIZATION MODEL

Variable	x_1	x_2	x_3	x_4	x_5
Optimal value	0	0	0.96	0.04	0
Variable	y_1	y_2	y_3	y_4	y_5
Optimal value	0	0.387	0.48	0	0.133

TABLE II. CHARACTERISTICS OF THE BEST NON-DOMINATED SOLUTION SELECTED BY USING THE BGFG

Discharger	**Treatment level (%)**	**Initial treatment cost $c(i)$ (million U.S. $)**	**Fuzzy Risk (%)**
1	0	0	48.9
2	51	0.301	
3	0	0	
4	95	0.131	
5	87	0.109	
6	51	0.038	
7	70	0.694	
8	85	0.205	

TABLE III. TREATMENT COST OF DISCHARGERS 2, 6 AND 7 PARTICIPATING IN DIFFERENT COALITIONS (MILLION U.S. $)

Coalition among dischargers	**Treatment cost of discharger**			**Cost reduction**
	2	*6*	*7*	
2, 6 and 7	0.224	0.130	0.493	0.184
2 and 6	0.256	0.063	-	0.02
2 and 7	0.301	-	0.694	0
6 and 7	-	0.104	0.527	0.1

TABLE IV. FINAL TREATMENT COST AND SIDE PAYMENTS OF DISCHARGERS IN COALITION OF DISCHARGERS 2, 6 AND 7 (MILLION U.S. $)

Discharger	**Share of each discharger from the total treatment cost reduction (allocation of cost reduction)**	**Reallocated (final) treatment cost**	**Side payment**
2	0.037	0.263	-0.039
6	0.037	0	0.13
7	0.110	0.584	-0.091
Sum	0.184	0.847	0

In this case, treatment cost of discharger 6 is more than its initial treatment cost and treatment costs of dischargers 2 and 7 are less than their initial treatment costs. Hence, dischargers 2 and 7 have to buy discharge permits from discharge 6. The total cost reduction, when dischargers 2, 6 and 7 form a coalition is equal to 0.184 million U.S. $.

In this paper, a cooperative game theoretic approach, namely Nucleolus game is utilized to equitably allocate the total cost reduction to dischargers and calculate side payments. The share of each discharger from the total cost reduction, final treatment cost after cost reallocation, and side payments are summarized in Table 4. In this table, a positive value for the side payment of discharger 6 shows that this discharger should sell discharge permit and gain money equal to its side payment. Similarly, negative values for the side payments of dischargers 2 and 7 show that these dischargers should buy discharge permits and pay money to discharger 6 equal to their side payments.

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