

# An ARMA Model for Natural Gas Consumption

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**Abstract.** In this study we propose a new model for modeling and forecasting natural gas consumption, which is important for efficient management of energy resources. The existing literature on modeling natural gas consumption uses time series models such as autoregressive moving average (ARMA) models. We extend the existing literature by proposing an ARMA(1,1) time series model with a Fourier series term for capturing the seasonality in the consumption process. Using the backtesting methodology, which is applied at monthly forecast horizon, we show that the proposed model performs well in forecasting the monthly per-consumer natural gas consumption.

**Keywords:** Natural gas consumption, ARMA model, Forecasting, Backtesting.

## 1. Introduction

Natural gas is an environment friendly fuel compared to fossil-based energy resources. The recent developments in the natural gas drilling technologies provide more incentives for increasing the share of the natural gas in the total energy mix of developed and emerging economies. As the importance of natural gas consumption increases, its accurate modelling and forecasting becomes more crucial for the efficient management of energy resources. Therefore, a significant amount of research has been conducted for better modelling of natural gas consumption.

A variety of models has been utilized to model natural gas consumption. An excellent literature review on natural gas modelling is provided by Soldo (2012). An important portion of these studies focus on time series modelling of natural gas consumption. Some of these studies include; Liu and Lin (1991), Crompton and Wu (2005), Ediger and Akar (2007), Aras and Aras (2004), Gumrah et al. (2001), Kumar and Jain (2010), Sarak and Satman (2003), and Erdogdu (2010). However, the literature on natural gas consumption modelling did not focus enough on the structure of the seasonality of natural gas consumption. In particular, natural gas consumption in residential and commercial sectors is mostly for space heating purposes and it is very dependent on the temperature conditions. Since temperatures are very seasonal, there exists strong seasonality in the natural gas consumption patterns at the daily and monthly levels.

In the study by Aras and Aras (2004) an ARMA model has been suggested and the sample is separated as heating and cooling months of the year. In the energy industry, heating degree days (HDDs) is defined as  $\max(18 - T_t, 0)$ , where  $T_t$  is the average temperature on  $t$ , whereas cooling degree days (CDDs) is defined as  $\max(T_t - 18, 0)$ . Heating/cooling degree days is used as an explanatory variable. However, the inclusion of heating/cooling degree days requires the introduction of a temperature model (e.g. see Goncu 2011, and Goncu et al. 2011 for stochastic temperature models) in order to forecast the future paths of natural gas consumption. In this study we capture the seasonality of the natural gas consumption via a Fourier series term added to the autoregressive moving average (ARMA) model, and thus there is no need to incorporate a temperature model. The residuals of the fitted model have no significant serial correlation and fits well to the normal distribution. Furthermore, results show that there is no significant serial correlation in the squared

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residuals of the fitted model, in other words there is no evidence of ARCH effect. Finally, we test the forecasting performance of our model via backtesting methodology.

## 2. Data

The dataset is obtained from IGDAS, which is the only natural gas distributor in Istanbul, Turkey. Our dataset contains daily observations of natural gas consumption in the residential and commercial areas of Istanbul, from January 1st, 2004 to October 18th, 2011. Daily observations are aggregated to obtain the monthly consumption values during the same period.

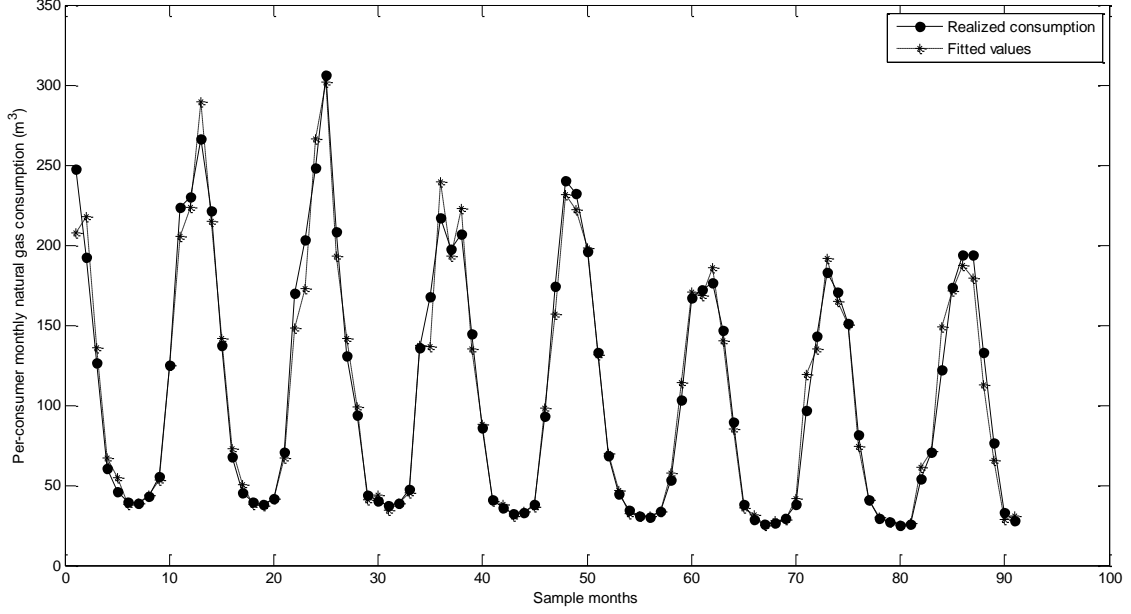


Fig. 1: Realized per-consumer monthly natural gas consumption versus the fitted values

In this study we prefer to model the per-consumer natural gas consumption since total natural gas consumption is dependent on the infrastructure investments. Natural gas distributor companies often have good information regarding the infrastructure investments and the number of consumers.

## 3. Model

We denote the natural logarithm of the monthly per-consumer natural gas consumption by  $Y_t = \ln(c_t)$ . The natural logarithm is used to guarantee the non-negativity of the natural gas consumption. The proposed model is given as

$$Y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \sum_{i=1}^P \alpha_i \sin(2\pi i t / 12) + \gamma_i \cos(2\pi i t / 12) + \phi Y_{t-1} + a_t - \theta a_{t-1} \quad (1)$$

where  $\{a_t\}$  is assumed as Gaussian white noise with mean zero and variance  $\sigma^2$ . As a result of the preliminary data analysis we set the number of sine and cosine coefficients as four, i.e.  $P=4$ . The  $R^2$  of the fitted model is 0.96. In Table 1, we present the estimates of the regression coefficients given in Equation (1). All coefficients are significant at the 90% level.

Table 1: Estimated coefficients of the regression model in Equation (1)

$\beta_0$	$\beta_1$	$\beta_2$	$\phi$	$\theta$
2.5077	-0.0053	0.00026	0.4691	0.2234
$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	
0.4176	0.0177	-0.0410	-0.0223	
$\gamma_1$	$\gamma_2$	$\gamma_3$	$\gamma_4$	
0.5077	0.0269	0.0763	-0.0253	

Monthly per-consumer natural gas consumption and the fitted model given in Equation (1) are plotted in Figure 1. The fitted model is also verified via the standardized residuals. In Figure 2, we plot the standardized residuals together with the histogram, whereas the sample autocorrelation function of the residuals and squared residuals are plotted in Figure 3. Figure 3 shows that there is no significant ARCH effect.

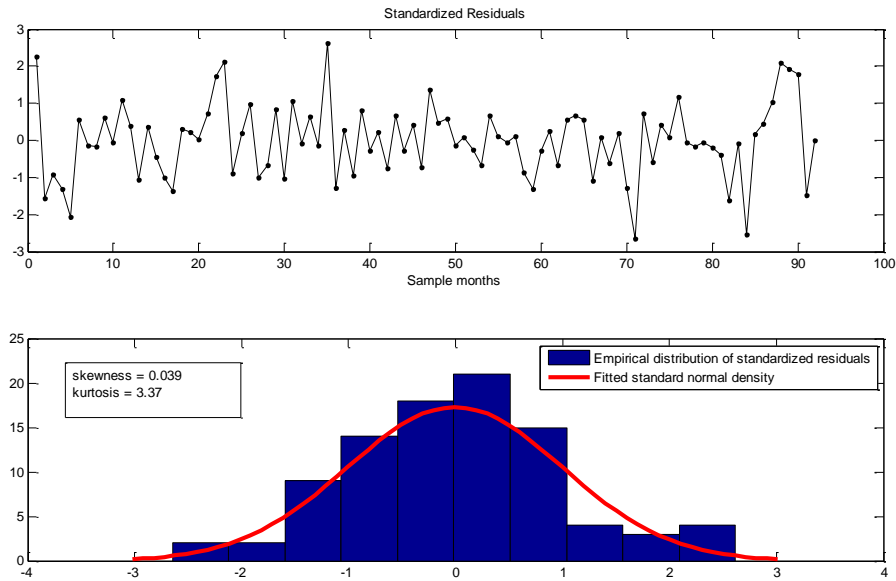


Fig. 2: Standardized residuals of the fitted model are plotted together with the histogram

#### 4. Forecasting

Forecasting performance of the model specified in Equation (1) is done via backtesting methodology. In this method we utilize the last 60 monthly observations to compare the model forecasts and realized consumption values. To start the backtesting process we use the first 32 monthly consumption values to estimate the model parameters and with the estimated parameters a forecast of the next month consumption is obtained. This value is compared with the realized consumption. For the next month, we expand estimation window by one more month and we re-estimate the model parameters and with the updated parameters we forecast the consumption for the next month.

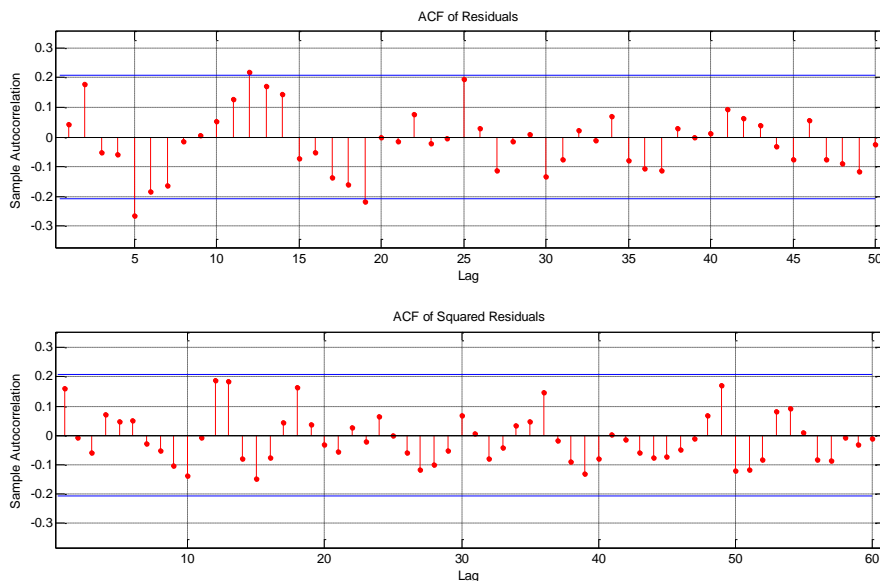


Fig. 3: Sample autocorrelation function of the residuals and squared residuals obtained from the fitted model

We repeat this procedure until all the forecasts are obtained within the backtesting sample. We plot the realized natural gas consumption versus the forecasted values with one month forecast horizon in Figure 4. The upper and lower bounds are obtained by adding and subtracting one standard deviation to the forecasts. To quantify backtesting results we calculate the relative mean square error (RMSE) as:

$$RMSE = \frac{1}{N} \sum_{t=1}^N \left( \frac{\hat{c}_{t+1} - c_{t+1}}{c_{t+1}} \right)^2, \quad (2)$$

where  $t = 1, 2, \dots, N$  denotes the number of observations used in the backtesting sample,  $\hat{c}_{t+1}$  represents the model forecast on month  $t$  with forecast horizon of 1 month and  $c_{t+1}$  is the actual natural gas demand at month  $t+1$ . We calculate the relative mean squared error of the model as 0.0747.

## 5. Conclusion

In this paper, we propose an ARMA model with trend and Fourier coefficients to capture the seasonality of the natural gas consumption. We apply our methodology to model and forecast monthly per-consumer natural gas consumption in Istanbul, Turkey. Our framework captures empirical properties of the monthly natural gas consumption. The fit of the model is quite good and the forecasting performance is verified via backtesting methodology at the monthly forecast horizon. The proposed model can be utilized for effective management of natural gas resources and distribution.

Some possible future research directions include the generalization of the ARMA model for non-Gaussian residuals. Alternatively, the fit of the seasonality function might be improved by the use of non-parametric estimation techniques.

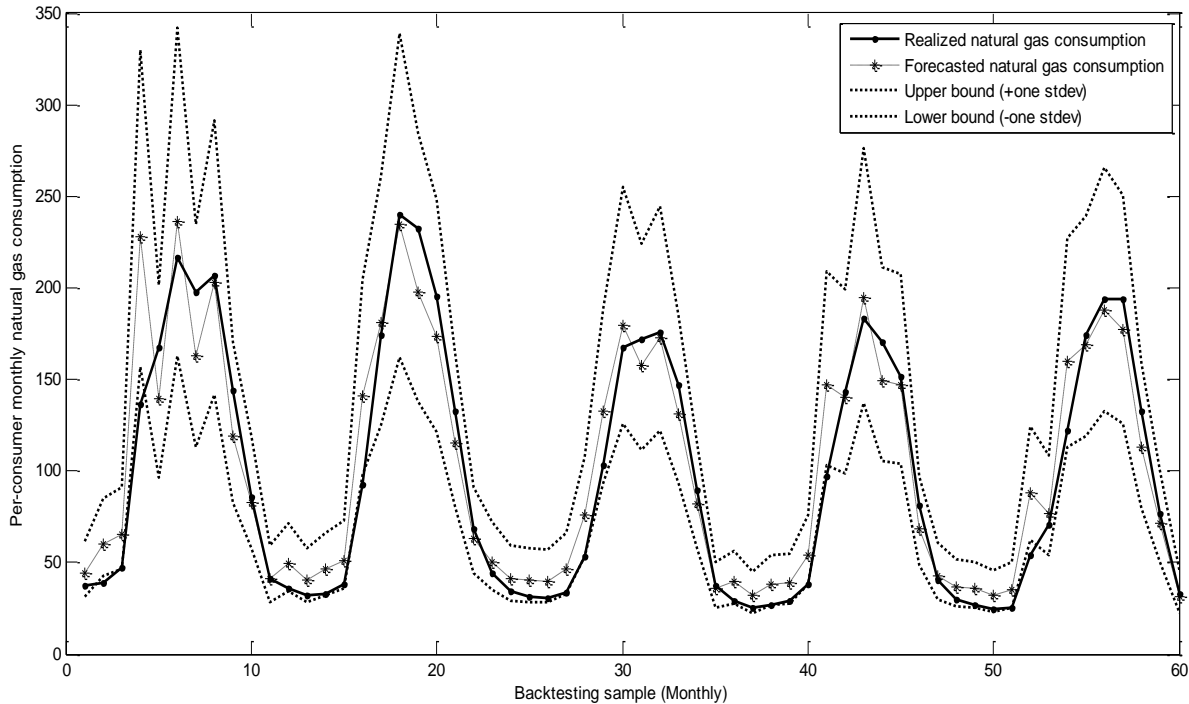


Fig. 4: Comparison of model forecasts with the realized monthly natural gas consumption values

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