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Pearson System Distribution Approximation in Wind Energy Potential Analysis

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Abstract. Wind energy is considered as one of the solution to the worldwide depletion of fossil fuel resources as well as the economic alternatives in protecting the atmosphere from the adverse consequences of global warming. Introducing of wind machine at a particular site however requires the knowledge of the distribution of the wind speed. The work presented in this paper considers the possibility of using Pearson system in approximation of wind speed distribution using the northern Borneo island wind speed data. Considering the stochastic properties of the wind speed, the density function approximation was obtained by considering the first to fourth statistical moments. The wind energy potential was evaluated based on the Pearson system frequency curve. Application of Pearson system has advantages in capability to take on variety shapes of distribution which makes it particularly applicable to the analysis of wind energy potential characterized by random wind speed data.

Keywords: Wind energy, Wind speed distribution, Pearson system, Density function approximation

1. Introduction

The threats of climate change caused by burning fossil fuels as well as escalating fossil fuel prices make the further rapid development of renewable energy sources a global imperative. Energy provided by the winds is considered as one of the renewable energy alternatives that meet the needs of modern societies in reducing the dependence on fossil fuels whilst at the same time delivering substantial reduction in greenhouse gas emissions. However introduction of winds as an alternative source of energy is often criticised for the reason that the power provided from the wind is intermittent and virtually uncontrollable. Considering the stochastic properties of wind speed, the assessment of power potential and the economic feasibility belonging to a particular site of wind machine are largely relies on knowledge of wind speed distribution [1]. Conventional statistical distributions used in fitting the measured wind speed data are generally based on the mean-variance analysis but it has been well suggested by numbers of research that the observed performance distributions are not always fully captured by the first two moments of the distribution [2]. The motivation for modelling skewness and kurtosis in the wind power production system has followed from attempts to understand the behaviour of a wide range of wind speed data based on various statistical moments. Cheng et al. [1], among others, have proposed that production costing and reliability of a wind power system is related to dynamic changes of skewness and kurtosis coefficients of the wind speed distribution. Given the needs in handling variety shapes of wind speed distribution, it has become interesting to propose frameworks that are flexible enough to accommodate distributions with broad range of properties. This paper is aimed to provide further insight into the wind energy potential analysis based on the wind speed distribution selected from the Pearson family.

The Pearson system is a parametric family of continuous probability distributions capable to model a wide scale of distributions with various skewness and kurtosis [3]. Firstly introduced by Karl Pearson in 1895 as an effort to model visibly skewed distribution, the Pearson family of distribution is amendable to both theoretical and empirical problems where density function must be explicitly expressed.

In this study, descriptive statistics of northern Borneo wind speed data was obtained from the meteorological data provided by Malaysian Meteorological Department (MMD) [4]. Data measured by Automatic Weather Stations located in Kota Kinabalu Sabah was selected in view of the low and unpredictable wind speed characterized by various monsoon of equatorial region. The density function approximation of the wind speed data was obtained using the Pearson system employing the first to fourth statistical moments. The wind power produced by the probabilistic mean wind speed was then evaluated considering the gearless blade tip wind turbine system.

2. Probabilistic Wind Energy Model

2.1. The Pearson System

The Pearson system is a parametric family of distributions used to model a broad scale of distributions with various third and fourth moments. This method of moments is a statistical technique to estimate probability distributions by equating their theoretical moments with the moments of empirical distributions. Pearson distributions are defined by a separable first order differential equation of

$$\frac{f'(x)}{x(x)} = \frac{P(x)}{Q(x)} = \frac{x-a}{b_0+b_1x+b_2x^2} \tag{1}$$

where f is a density function and a, b_0, b_1 and b_2 are the parameters of the distribution. An important characteristic of the Pearson system is the direct correspondence between the parameters and the first four moments (μ_1, ..., μ_4) of the distribution [5]. The expressions for the moments are obtained by solving Equation 1 using integration by part. The expressions for the parameters a, b_0, b_1 and b_2 in terms of the first four moments can be obtained from the set of Equation (2).

$$\begin{aligned} b_1 = a &= -\frac{\mu_3\left(\mu_4+3\mu_2^2\right)}{A} = -\frac{\mu_2^{1/2}\beta_1\left(\beta_2+3\right)}{A'} \\ b_2 &= -\frac{\left(2\mu_2\mu_4-3\mu_3^2-6\mu_2^3\right)}{A} = -\frac{\left(2\beta_2-3\beta_1^2-6\right)}{A'} \\ b_0 &= -\frac{\mu_2\left(4\mu_2\mu_4-3\mu_3^2\right)}{A} = -\frac{\mu_2\left(4\beta_2-3\beta_1^2\right)}{A'} \end{aligned} \tag{2}$$

where the two moment ratios

$\beta_1^2 = \mu_3^2 / \mu_2^3$ and $\beta_2 = \mu_4 / \mu_2^2$

respectively denote skewness and kurtosis. The scaling parameters A and A'are obtained from

$$\begin{aligned} A &= 10\mu_4\mu_2 - 18\mu_2^3 - 12\mu_3^2 \\ A' &= 10\beta_2 - 18 - 12\beta_1^2 \end{aligned}$$

Classification and selection of the distribution are achieved by finding the roots of the denominator of quadratic Equation 1 and the Pearson's coefficient given by

$$K = \frac{b_1^2}{4b_0b_2} \tag{3}$$

Depending on the values of these parameters, different types of Pearson curves can be obtained such that

1.If $K < 0$, roots are real and of opposite signs. This corresponds to beta distribution or type-I distribution in the Pearson system.

2.If K > 1, roots are real and have the same sign. This corresponds to beta distribution of the second kind, or type-VI distribution.

3.If 0 < K < 1, roots are complex. This corresponds to type IV-distribution.

Pearson proposed further class distinctions by taking into account certain distributions and boundaries between classes and classified the solutions into types numbered 1 to 12 [2].

2.2. Descriptive Statistics of Wind Speed Data

Located on the west coast of Sabah on the northern portion of the island of Borneo, the data sets of this study consist of daily wind speed measured in meter per second. Measured by the MMD at the unmanned weather observation station situated in Kota Kinabalu, the data consists of monthly wind speed for the year of 2009. The descriptive statistics for the data set is shown in Table 1.

	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Mean	1.958	1.859	2.419	2.018	2.033	2.242	2.406	2.492	2.714	2.544	2.005	1.842
Std. Dev.	0.982	0.925	1.129	1.035	1.084	1.148	1.130	1.389	1.513	1.351	0.994	0.807
Skewness	0.552	0.490	0.435	0.559	0.689	0.449	0.598	0.842	0.650	0.630	0.609	0.461
Kurtosis	3.131	2.863	2.852	3.045	3.242	2.714	3.143	3.590	3.319	2.968	3.068	3.026
Sample Size	744	672	744	720	744	720	744	744	720	744	720	744

Table 1: Monthly Descriptive Statistics

Kota Kinabalu features a tropical rainforest climate which characterized by the Northeast and the Southwest Monsoon [6]. The Northeast Monsoon occurs between November and March while the Southwest Monsoon occurs between May and September. As shown in Table 1 the mean wind speed recorded during the Southwest Monsoon is higher than mean wind speed recorded during the Northeast Monsoon. The descriptive statistics also indicated that the highest mean wind speed was recorded in September which is the transition period from the Southwest to Northeast Monsoon.

2.3. Density Function Approximation

Potential indices of a particular site for a wind energy project such as the mean wind power are typically expressed as single numerical values. These numerical indices provide the expected values of the distributions that fully describe the particular indicator being expressed. The expected value of a distribution is one of its several parameters, which provide a complete mathematical description of the distribution function. This mean value alone does not, however, provide any information on the variability of the index being described. Therefore, a potential wind energy index of a particular site is best represented as a probability distribution, or probability density function, PDF [1]. In this study, probability distribution of the wind speed was estimated using the Pearson system. Based on the first through fourth statistical moments, the type of the Pearson system was determined by Equation 4 and the quadratic roots which represent the Pearson's criterion for fixing the distribution family as shown in Table 2.

	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Coeff. K	-0.19	-0.12	-0.10	-0.17	-0.27	-0.09	-0.21	-0.48	-0.28	-0.18	-0.19	-0.14
a_1	7.55	5.03	5.31	6.15	6.47	4.24	6.81	7.57	8.71	4.78	5.76	7.39
a_2	-2.36	-2.14	-2.36	-2.16	-1.90	-2.06	-2.14	-1.71	-2.19	-1.78	-1.98	-2.65

Table 2: Pearson's Criterion for Fixing the Distribution Family

Taking into account that the Pearson's coefficient K < 0 and the roots are real and opposite sign the distribution is classified as beta distribution or Pearson's type 1. The frequency curve for the type 1 Pearson's family of distribution for the wind speed is represented by:

$$f(x) = k\left(1+\frac{x}{a_1}\right)^{m_1}\left(1-\frac{x}{a_2}\right)^{m_2} \tag{4}$$

where k is the normalization constant.

Integrating the frequency curve the values of percentage points in the distribution can represented by the cumulative density function, CDF. Figure 1 shows PDF and CDF of the wind speed approximated using the Pearson system. Referring to cumulative density function, the probabilistic mean of the wind speed is represented in Table 3.

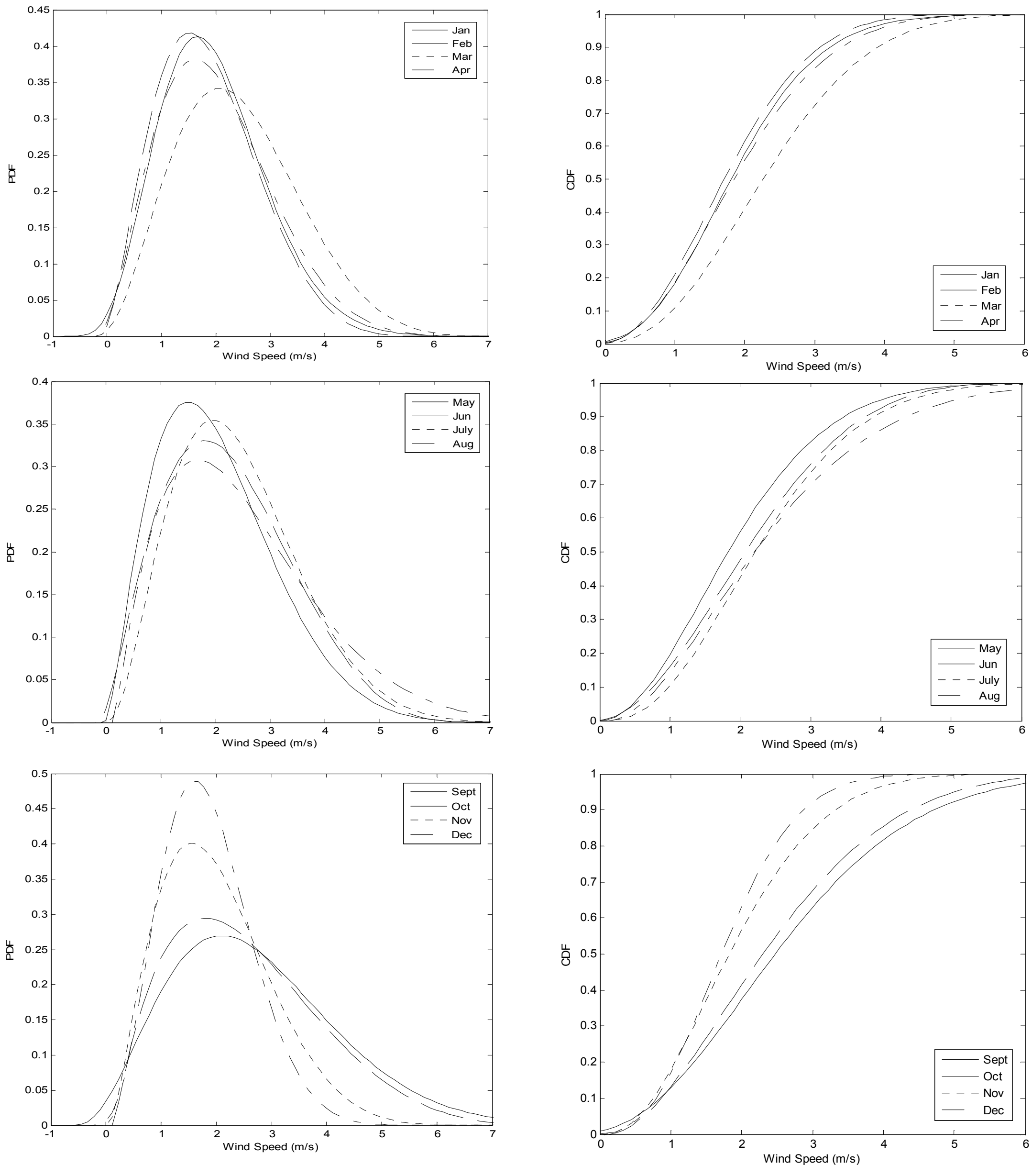

Figure 1: PDF and CDF of the wind speed

Considering the gearless blade tip wind turbine with the blade diameter, D of 1.8 meter and the probabilistic mean wind speed, v the available power in the wind by was evaluated by the equation of

$$P_{Wind} = \rho \times \pi \left(\frac{D}{2}\right)^2 \times v^3 \tag{5}$$

where ρ represents the density of the air approximated at $1.2 kg/m^3$. Figure 2 shows the mean of available power in the wind for the 12 months periods.

	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Power	1.810	1.710	2.266	1.850	1.830	2.073	2.219	2.205	2.470	2.304	1.826	1.717

Table 3: Probabilistic mean of the wind speed

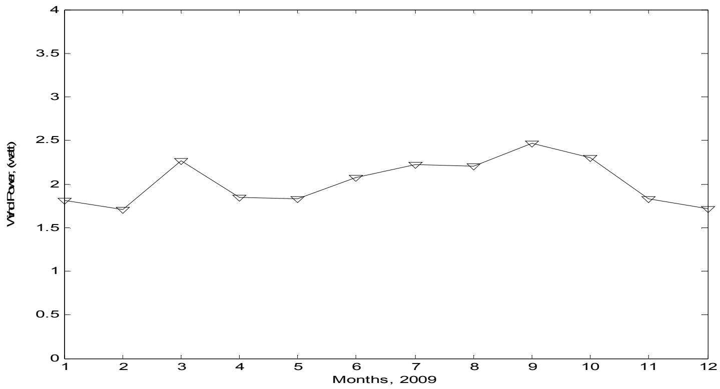

Figure 2: Available power in the wind

3. Conclusion

Conventional statistical distributions used in fitting the measured wind speed are largely relies on the mean-variance analysis assuming a symmetric distribution. However it has been well suggested by numbers of research that the observed performance distributions are not always fully captured by the first two moments of the distribution. The use of the Pearson curve in approximating the probability distribution of wind speed has the advantages in terms of capability to take on variety shapes of distribution which makes it particularly applicable to the analysis of wind energy potential characterized by random wind speed data.

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