

Transport Electron through a Quantum Wire by Side-Attached Asymmetric Quantum-Dot Chains

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Abstract—Transport electron at zero temperature through a quantum wire with side-attached two asymmetric quantum dot arrays is theoretically studied using the non-interacting Anderson tunneling Hamiltonian method. We show that the structure of QD-chains strongly influences the amplitude and spectrum of quantum wire transmission characteristics. It is shown that whenever the number of chains-quantum dots is balance the number of forbidden mini-bands in quantum wire conductance is the same number of quantum dots in every chain, but in unbalance case the number of mini gap increases to the sum of the number of quantum dots in two chains and we have new anti-resonance and resonance points in spectrum. Considering the suitable gap between QD-chains can strengthen the amplitude of new resonant peaks in QW conductance spectrum. The proposed asymmetric-QD scatter system idea in this paper opens a new insight on controlling electron conductance through a quantum wire nanostructures.

Keywords—Asymmetric quantum dot chains, scatter system, electron transport, quantum wire

I. INTRODUCTION:

The progresses in nanofabrication technology have allowed the study of electron transport and especially the conductance through the quantum nano-scale systems likes quantum wires (QW) and quantum dots chains (QD) -in a very controllable way - which are very interesting from nano-electronics applications point of view [1, 2]. In these systems, the samples dimensions are smaller than the coherence length characterizing the single electron wavefunction and the phase coherence implemented in this manner leads to interesting interference effects [3]. Quantum effects in rings attached to leads, quantum wire connected to quantum dots and multi-chain nano-rings, serve as promising prototypes to the design of further nano-devices, since coupling to the continuum states shows an even-odd parity effect in the conductance when the Fermi energy is localized at the center of the energy band [3-9].

The systems such as uniform QD array, nano-wires and nano-ring [10-14] which are side-coupled to a quantum wire act as a scatter system for electron transmission through the QW and have a major effect on electronic conductance of nano-device. For a uniform QD-chains array with M sites, it was shown that the transmission characteristic has M anti-resonances and M-1 resonance, M mini-gaps and M-1

allowed mini-bands arise [10]. Consequently, it should be stressed that the suggested particular setups, allows us to control the energy and width of the anti-resonances in an independent fashion.

In this paper, we study theoretically electron transport properties through a quantum wire (QW) system with side-attached asymmetric QD arrays (as a scatter system). The effect of dots number in the balanced and unbalanced QD-chains and also distance between QD-chains on electronic conductance are evaluated. It is shown that the asymmetric QD-scatter system allows us to manipulate the spectrum and amplitude of QW conductance characteristic. In section II, the proper model for calculation of the conductance through QW with asymmetric scatter system by using a non interacting Anderson tunneling Hamiltonian model is presented. Simulation results and discussion are illustrated in Section III. Finally, the paper is completed with a brief conclusion.

II. ASYMMETRIC SCATTER STRUCTURE AND MODEL DERIVATION

Figure 1 shows a general form of two asymmetric quantum dot-chains as a scatter system for manipulating of electron transport through a quantum wire. This asymmetry may involve some disordering such as considering the unbalance QD-chains and gap between two chains. We assume first chain contain N number of QDs and second chain have M number of QDs. Parameter of p indicate the distance between chains. The system assumed in equilibrium, is modeled by using a non interacting Anderson tunneling Hamiltonian [10] which can be written as:

$$H = H_{QW} + H_{QM} + H_{QW-QM} \quad (1.a)$$

where

$$H_{QW} = \vartheta \sum_{j=-\infty}^{+\infty} a_j^\dagger a_{j+1} + h.c \quad (1.b)$$

$$H_{QCH} = \varepsilon_0 \sum_{n=1}^N d_n^\dagger d_n + V_c \sum_{n=1}^{N-1} d_n^\dagger d_{n+1} + h.c \quad (1.c)$$

$$+ \varepsilon_0 \sum_{m=1}^M c_m^\dagger c_m + V_c \sum_{m=1}^{M-1} c_m^\dagger c_{m+1} + h.c$$

$$H_{QCH-QW} = V_0(d_1^\dagger a_1 + h.c. + c_1^\dagger a_\beta + h.c.) \quad (1.d)$$

Here, H_{QW} describes the dynamics of the QW, ϑ being the hopping between neighbor sites of the QW, and a_j^\dagger (a_j) creates (annihilates) an electron in the j th site. H_{QCH} is the Hamiltonian of the asymmetric QD-scatter system d_n^\dagger (d_n) and c_m^\dagger (c_m) are the creation (annihilation) operators of an electron in the quantum dots n and m of the first and second QD-chain respectively. ε_0 is the corresponding single level energy and V_c the tunneling coupling between sites in the quantum dots in every chains assumed all equal. H_{QCH-QW} is the coupling of the QW with the asymmetric QD-chains. However H_{QCH-QW} may be changed for different configuration of QD-scatter system (is explained in the following in detail).

The quantum wire Hamiltonian H_{QW} corresponds to the free-particle Hamiltonian on a lattice with spacing d and whose eigenfunction can be expressed as Bloch solutions [11]

$$|k\rangle = \sum_{j=-\infty}^{+\infty} e^{ikjd} |j\rangle \quad (2)$$

Where $|k\rangle$ is the momentum eigenstate and $|j\rangle$ is a wannier state localized at site j . The dispersion relation associated with these Bloch states is [11]

$$\varepsilon = 2\vartheta \cos kd \quad (3)$$

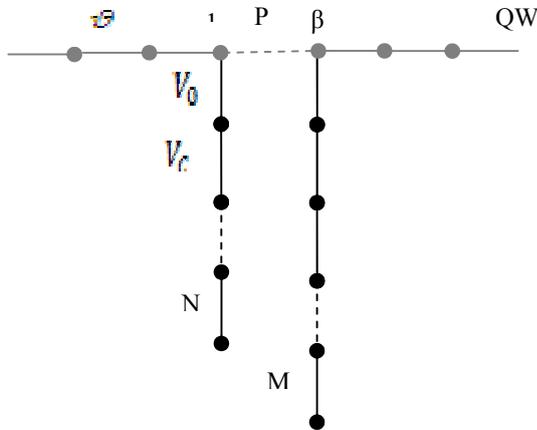


Figure 1. Side-coupled asymmetric quantum dot chains as a scatter system attached to a perfect quantum wire.

The stationary states of the entire Hamiltonian H can be written as:

$$|\varphi\rangle = \sum_{j=-\infty}^{+\infty} a_j^k |j\rangle + \sum_{n=1}^N d_n^k |n\rangle + \sum_{m=1}^M c_m^k |m\rangle \quad (4)$$

Where the coefficient a_j^k (d_n^k and c_m^k) is the probability amplitude to find the electron in the site j of the QW (in the quantum dots n and m of the first and second chain

respectively) all in the state k . The amplitudes, a_j^k , d_n^k and c_m^k obey the following linear difference equations:

$$\varepsilon a_j^k = \vartheta(a_{j-1}^k + a_{j+1}^k) \quad j \in \langle 1, j \rangle \beta \quad (5.a)$$

for $0 \langle j \langle \beta$:

$$\varepsilon a_j^k = \vartheta(a_{j-1}^k + a_{j+1}^k) + V_0 d_1^k \quad j = 1 \quad (5.b)$$

$$\varepsilon a_j^k = \vartheta(a_{j-1}^k + a_{j+1}^k) + V_0 c_1^k \quad j = \beta \quad (5.c)$$

$$\varepsilon d_1^k = \varepsilon_0 d_1^k + V_c d_2^k + V_0 a_1^k \quad n = 1 \quad (5.d)$$

$$\varepsilon c_1^k = \varepsilon_0 c_1^k + V_c c_2^k + V_0 d_\beta^k \quad m = 1 \quad (5.e)$$

$$\varepsilon d_n^k = \varepsilon_0 d_n^k + V_c (d_{n-1}^k + d_{n+1}^k) \quad (5.f)$$

$$n = 2, 3, \dots, N$$

$$\varepsilon c_m^k = \varepsilon_0 c_m^k + V_c (c_{m-1}^k + c_{m+1}^k) \quad (5.g)$$

$$m = 2, 3, \dots, M$$

$$\varepsilon d_N^k = \varepsilon_0 d_N^k + V_c d_{N-1}^k \quad n = N \quad (5.h)$$

$$\varepsilon c_M^k = \varepsilon_0 c_M^k + V_c c_{M-1}^k \quad m = M \quad (5.i)$$

The amplitudes d_n^k and c_m^k can be calculated in term of a_j^k in iterating backwards method:

$$a_1^k = \frac{V_0^2 a_j^k}{\varepsilon - \varepsilon_0 - \frac{V_c^2}{\varepsilon - \varepsilon_0 - \frac{V_c^2}{\varepsilon - \varepsilon_0 - \frac{V_c^2}{\varepsilon - \varepsilon_0 - \dots}}}} \quad (6.a)$$

$$c_1^k = \frac{V_c^2 a_j^k}{\varepsilon - \varepsilon_0 - \frac{V_c^2}{\varepsilon - \varepsilon_0 - \frac{V_c^2}{\varepsilon - \varepsilon_0 - \frac{V_c^2}{\varepsilon - \varepsilon_0 - \dots}}}} \quad (6.b)$$

where

$$Q_1 = \varepsilon - \varepsilon_0 - \frac{V_0^2}{\varepsilon - \varepsilon_0 - \frac{V_c^2}{\varepsilon - \varepsilon_0 - \frac{V_c^2}{\varepsilon - \varepsilon_0 - \dots}}} \quad (6.c)$$

$$Q_2 = \varepsilon - \varepsilon_0 - \frac{V_c^2}{\varepsilon - \varepsilon_0 - \frac{V_c^2}{\varepsilon - \varepsilon_0 - \frac{V_c^2}{\varepsilon - \varepsilon_0 - \frac{V_c^2}{\varepsilon - \varepsilon_0 - \dots}}} \quad (6.d)$$

Which Q_1 and Q_2 depend on the number of N and M respectively [10].

For the calculation of transmission equation, it is assumed that the electrons are described by a plane wave

incident from the far left of QW with unity amplitude and a reflection amplitude r and at the far right of QW by a transmission amplitude, t . Therefore we can write:

$$a_j^k = e^{ikdj} + r e^{-ikdj} \quad j \langle 1 \quad (7.a)$$

$$a_j^k = t e^{ikdj} \quad j \rangle \beta \quad (7.b)$$

Finally transmission amplitude can be calculated by inserting a_j^k into corresponding equations which leads to an inhomogeneous system of linear equations for t , as follows:

$$t = \frac{i \sin kd}{i \sin kd + X} \quad (8)$$

X will be changed for different configuration of QD-scatter system. In the following we model the scatter system from QD-arrays gap and unbalance point of view.

A. Gap between chains:

In this section, the effect of gap (distance) between QD-chains is considered, that is, if the first QD-chain was connected to l th site in QW, the second QD-chain is connected to $j+p$ th site. In the following, two cases for the value of p are considered and the corresponding transmission coefficients are calculated.

Cases 1: $p=1$.

In this case, based on the scatter system configuration (as it was mentioned before), the H_{QW-QW} parameter β are calculated as:

$$\beta = 3 \quad (9.a)$$

and X will be:

$$\begin{aligned} X &= \frac{V_0^2}{2\vartheta Q_1} + \frac{V_0^2}{2\vartheta Q_2} - \frac{V_0^4 e^{ikd}}{4\vartheta^2 Q_1 Q_2} - \frac{V_0^4 e^{3ikd}}{4\vartheta^2 Q_1 Q_2} \\ &= \frac{\mu}{Q_1} + \frac{\mu}{Q_2} - \frac{\mu^2 e^{ikd}}{Q_1 Q_2} - \frac{\mu^2 e^{3ikd}}{Q_1 Q_2} \end{aligned} \quad (9.b)$$

Where $\mu = \frac{V_0^2}{2\vartheta}$.

Now we insert the equation (9.b) into the equation (8) and then calculate the transmission coefficient. The transmission probability is given by $T = |t|^2$ and it is related to the linear conductance G at the Fermi energy ε by the one-channel Landauer formula at zero temperature as [15-16]:

$$G(\varepsilon) = \frac{2e^2}{h} T(\varepsilon) \quad (10)$$

Cases 2: $p=2$.

If we set $p=2$, it is shown that, the calculated transmission coefficient will be different. The similar calculation approach can be done:

$$\beta = 4 \quad (11.a)$$

$$X = \frac{\mu}{Q_1} + \frac{\mu}{Q_2} - \frac{\mu^2 e^{ikd}}{Q_1 Q_2} - \frac{\mu^2 e^{3ikd}}{Q_1 Q_2} - \frac{\mu^2 e^{5ikd}}{Q_1 Q_2} \quad (11.b)$$

B. Unbalanced number of dots :

Let us consider the effect of asymmetry in the unbalance QD-chains.

Cases 1: $M=N$.

As it is seen from Figure 1, we assume the number of QD in first chain is N and in the second chain is M . In this part we calculate X for the balance chains ($Q_1 = Q_2$). Therefore we obtain the following equation:

$$X = \frac{2\mu}{Q} - \frac{\mu^2 e^{ikd}}{Q^2} \quad (12)$$

Cases 2: $M \neq N$.

In this case, if the first chain have N site the second one have M site quantum dots so we can find the transmission parameter (X) as below:

$$X = \frac{\mu}{Q_1} + \frac{\mu}{Q_2} - \frac{\mu^2 e^{ikd}}{Q_1 Q_2} \quad (13)$$

III. SIMULATION RESULTS AND DISCUSSION:

In this section, simulated results including the electronic conductance through quantum wire with side-coupled asymmetric QD-arrays as a scatter system are presented and discussed. Effect of dots number in QD-chains on quantum wire transmission for two balanced and unbalanced cases is illustrated in Figure 2.a and 2.b. In this figure, the dimensionless conductance ($g = G/(2e^2/h) = T$) is plotted versus Fermi energy in units of μ (ε/μ). In balanced case(2.a) ($N=M$), the number of forbidden mini-bands is exactly the same as the number of quantum dots in chains ($N=M$) and increasing the number of quantum dots increases the number of anti-resonances so the system develops a set of alternating forbidden and allowed mini-bands in the range $[-V_c; V_c]$. If the number of quantum dots in each chain is odd (even), an anti-resonance (resonance) is observed at $\varepsilon=0$. However in unbalanced case (2.b), the number of forbidden mini-bands is equal to the sum of the number of quantum dots in two chains ($N+M$) and the system always has an anti-resonance (resonance) at $\varepsilon=0$, if this sum is odd (even). In this asymmetric case the electronic conductance contains rich spectral properties; as an example, the new anti-resonances appear due to different arrangements of quantum dots in two chains.

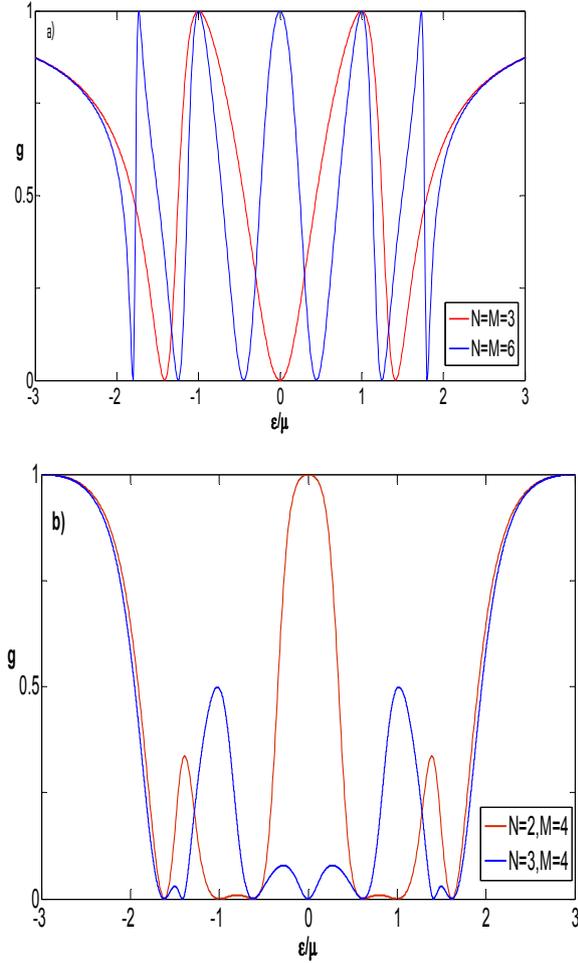


Figure 2. Dimensionless conductance versus Fermi energy in units of μ and $p=0$ in two-chains QD with $V_C = \mu$ and $\epsilon_0 = 0$, for a) balance chains b) unbalance chains.

Figure 3 shows the dimensionless conductance for cases we considered gap between two QD-chains ($p=1$ and 2). In Figure 3.a, $N=3$ and $M=4$ are set for first and second chains. It is clear that the sum of dots in two-chains ($N+M$) determines the number of anti-resonances in conductance spectrum and it is independent of the value of p . In other words, the number of mini gaps matches exactly the number of QDs chains. As shown in this figure, the amplitude of conductance is different for two values of (p) in such a manner this amplitude is larger for $p=1$ than $p=2$. The situation is same for without-gap and $p=2$ cases and amplitude of conductance is larger for $p=1$ case is due to coherence constructive-interference in the ballistic channel occurs for the electron stationary states in this case. Therefore, considering the suitable gap between QD-chains can increase the amplitude of QW conductance peaks..

It should be noted that the location of anti-resonance points are the same for two cases. If we consider the number of dots in two-chains same ($N=M=5$), the number of anti-

resonances will be equal with the number of dots and we have observed three anti-resonances in the conductance spectrum (figure 3.b).

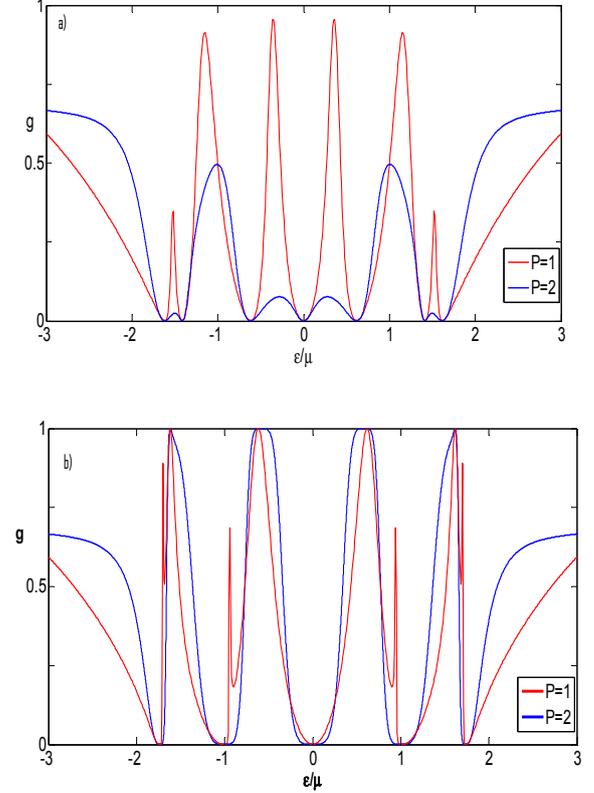


Figure 3. Dimensionless conductance versus Fermi energy in units of μ , for two-chains for a) $N=3$ and $M=4$ and b) $N=M=5$, with $V_C = \mu$ and $\epsilon_0 = 0$, for two cases of $p=1$ and $p=2$.

IV. CONCLUSION:

In this paper, a description of effect of the asymmetric quantum dot chains as a scatter system on electronic transport through a quantum wire has been studied using a non interacting Anderson tunneling Hamiltonian model. It was found that the transmission probability displays a different amplitude and spectrum for different configuration of QD-chains scatter system. When the number of quantum dots in chains was considered unbalance, the number of forbidden mini-bands increases with respect to balance case. Also it was observed that whenever we considered the suitable gap between QD-chains, the amplitude of quantum wire conductance could be enhanced in resonant points.

As a final result, the particular setup which we suggested, allows us to manipulate the spectrum and amplitude of QW-nanostructure conductance in an independent fashion. Finally, the proposed QW- system by asymmetric side-attached QD-chains can be used as the basic cell in design of new resonant-tunneling (RT) electronic devices.

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