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Simulation of River Flow Using Fourier Series Models

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Abstract. Rivers are the major portion of water resources in the world. In order to Management of water resources such as river basin, using new applicable methods is a necessity. So, modeling and simulation of river flow time series is an important step in the planning and operational analysis of water resources systems Forecasting of hydrologic systems by using the date of the system is one of the most important advantages of time series (dynamic systems) in water sciences. By using time series analysis that is based on mathematical logics and statistical solutions and recent electronic solution methods, it is possible to evaluate the system's reactions in advance by using the systems past behavior characteristics. In most water resource systems design and operation studies, the periodic phenomena have been represented by Fourier functions. Fourier analysis has become a standard tool in any hydrologic study concerning periodicity because Fourier analysis and modeling present powerful tools to analyze different periodic events behavior. For analysis and design of water resource systems and river basin management, it is sometimes useful to generate high- resolution synthetic river flows. Thus, this paper compares three different Fourier based models in their capabilities and results. These three models are: Fourier PARMA models (F-PARMA), Adapted Fourier analysis with kalman filter (AFAM), Fourier series ARIMA model (FSAM). These models test on “Khersan” river flow. It is prospected that the results show the best way and its reliability.

Keywords: Water resources, river flow, Fourier analysis, Fourier transforms.

1. Introduction

Numerous factors contributed in environmental changes which ultimately resulted in creation of variable environs morphologically as well as applicably. Among the most vital factors is erosion which plays a crucial role in appearance and land use changes. Significant sorts of erosion include wind erosion, hydro-erosion as well as erosion due to human applications. Water with a surprising power is a key component in erosion and sedimentation riverbeds and coastal lines. Moreover, valleys and vast plains are formed due to water erosion which is mainly associated with water flow. To measure the water flow a quantity called “discharge” is applied. Thus, study of river erosion is carried out based on information about water flow (discharge measurement) in combination with geological properties. In this regard, investigation of collected data which provide the time series of river discharge is considered as a rational and applicable method to predict the future flow values. A natural river flow process has significant periodic behavior in mean, standard deviation, skewness, and serial dependence structure.

Afshar and Fahmi (1996) provided a model to predict the rainfall in Iran by combining Fourier model with ARIMA models. Classic auto regressive-Moving average models introduced by Box and Jenkins in 60 decade and improved by Davis and Brackwell (1978). According to Yevjevich (1972), hydrologic time series

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can be modeled by a deterministic component and a stationary stochastic process. The deterministic component is composed of trends, jumps, and periodicities, while the stochastic component consists of the random deviations of the seasonal values (Haan 1982). This type of analysis has been applied to many kinds of hydrologic time series including water-use time series and groundwater table time series (Law 1974; Rao et al. 1975; Knotters and Van Walsum 1997).

Sedghi (1999) used integrated auto regressive moving average models to study and modeling of Karoon river discharge data and their future variations. Zare and Sedghi (2009) analyzed Shahar-chaii river discharge data in frequency domain and fitted a fourier model to time series of discharge data.

This study aims to investigate and compare three conventional methods applied to measure flow discharge based on analysis of time series which include: 1.Fourier Series ARIMA models (FSAM) 2.Adapted Fourier analysis with "kalman" filter (AFAM) 3.Fourier PARMA models. These models test and compare on natural river flow.

2. Fourier series ARIMA models (FSAM)

Mostly, it is easier to show a function by a set of primary functions called as base. So that it could be possible to illustrate the whole studied functions as linear composites of primary functions in base. Among the applicable function are (*sin)* as well as (*cos)* functions or mixed indices directly applicable to frequency analysis. In 1976, the studied model was applied for the first time by "Bloomfield" to analysis the time series in hydrology [1]. Fourier model is a mathematical structure designed of Fourier equation in combination with "Markov model".

2.1. Fit pattern

Fit pattern is represented as follow [1]:

$$X_{n,t} = \mu_t + \delta_t \xi_{n,t} \tag{1}$$

Where, μ_t and δ_t are average coefficient and standard deviation of harmonic series, respectively which are computed by following equation:

$$\mu_t = m_x + \sum_{j=1}^{\omega}(A_j Cos\frac{2\pi j}{\omega}t + B_j Sin\frac{2\pi j}{\omega}t) \tag{2}$$

Where, ω is series frequency period.

To compute the February series expansion coefficient (B_j and A_j), following equation is used:

$$A_j = \frac{2}{\omega}\sum_{j=1}^{\omega} m_j Cos\frac{2\pi j}{\omega}t \tag{3}$$

$$B_j = \frac{2}{\omega}\sum_{j=1}^{\omega} m_j Sin\frac{2\pi j}{\omega}t \tag{4}$$

In the above relation, m_t is average of inputs in a distinct interval time (month) during the statistical period and m_x is the total average of inputs. Standard deviation of harmonic series, δ_t, could be measured as:

$$\delta_t = S_t + \sum_{j=1}^{\omega}(A'_j Cos\frac{2\pi j}{\omega}t + B'_j Sin\frac{2\pi j}{\omega}t) \tag{5}$$

Where,

$$A'_j = \frac{2}{\omega}\sum_{j=1}^{\omega} S_t Cos\frac{2\pi j}{\omega}t \tag{6}$$

$$B'_j = \frac{2}{\omega}\sum_{j=1}^{\omega} S_t Sin\frac{2\pi j}{\omega}t \tag{7}$$

S_t = standard deviation of inputs in a distinct interval time (month) during statistical period

3. Adapted Fourier analysis with Kalman filter (AFAM)

AFAM model provided on the basis of a complicated mathematical structure has been simulated using statistic, mathematic as well as electronic sciences to predict the dynamic systems. "Zeki chen" (1980) attentively simulated discharge of flow in "Guta" and "Colombia" rivers and then predicted the future levels for the studied rivers. This model then called as kalman model:

$$\hat{Y}(i|i-1) = \phi(i,i-1)\hat{Y}(i-1|i-1) \quad (8)$$

$$P(i|i-1) = \phi(i,i-1)P(i-1|i-1)\phi(i,i-1) + Q(i) \quad (9)$$

$$K(i) = P(i|i-1)H^T(i)\left[H(i).P(i|i-1)H^T(i) + R(i)\right]^{-1} \quad (10)$$

$$\hat{Y}(i|i) = \hat{Y}(i|i-1) + K(i)\left[Z(i) - H(i)\hat{Y}(i|i-1)\right] \quad (11)$$

$$P(i|i) = \left[I - K(i)H(i)\right]P(i|i-1) \quad (12)$$

There are three essential requirements in the model designed on the basis of kalman filter model: state variable vector, 2-Rule of transfer the state variable from one time to a further time and 3-Primary state vector. In this model, Fourier equation is used as mode variable vector. In other words, the considered equation is the simulator function for real values. To design the model according to mentioned above analysis, by computation of $x(i) - x(i-1)$, following equation is achieved:

$$x(i) = x(i-1) + \sum_{k=1}^{m}\left(A_k(i-1)[\sin(\gamma ki) - \sin\{\gamma k(i-1)\}] + B_k(i-1)[\cos(\gamma ki) - \cos\{\gamma k(i-1)\}]\right) + \omega(i) \quad (13)$$

$$\alpha_k = \sin(\gamma ki) - \sin(\gamma k(i-1)), \beta_k = \cos(\gamma ki) - \cos(\gamma k(i-1)) \quad (14)$$

$$x(i) = x(i-1) + \sum_{k=1}^{m}\left[A_k(i-1)\alpha_k + B_k(i-1)\beta_k\right] + \omega(i) \quad (15)$$

$$\begin{bmatrix} x(i) \\ M(i) \\ A_1(i) \\ B_1(i) \\ A_2(i) \\ B_2(i) \\ \vdots \\ A_m(i) \\ B_m(i) \end{bmatrix} = \begin{bmatrix} 1 & 0 & \alpha_1 & \beta_1 & \alpha_2 & \beta_2 & \cdots & \alpha_m & \beta_m \\ 0 & 1 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix} . \begin{bmatrix} x(i-1) \\ M(i-1) \\ A_1(i-1) \\ B_1(i-1) \\ A_2(i-1) \\ B_2(i-1) \\ \vdots \\ A_m(i-1) \\ B_m(i-1) \end{bmatrix} + \begin{bmatrix} \omega(i) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Mathematical illustration of above matrix is: $Y(i) = \phi(i,i-1).Y(i-1) + W(i)$ (16)

4. Fourier-Parma models (F-PARMA)

In the area of stochastic hydrology, standardizing or filtering is used to transform periodic time series to stationary series before fitting stationary stochastic models but standardizing or filtering of most river flow series will not yield stationary residuals due to periodic autocorrelations. In these cases, the resulting model is miss specified [17]. To model such periodicity in autocorrelations, periodic autoregressive moving average (PARMA) models can be employed. In most cases, PARMA models have been applied to time series at a time scale of a months or more. However, when the number of periods is large (e.g, weekly data), PARMA models estimation of an exorbitant number of parameters, hereto for making PARMA modeling virtually impractical [18]. The parsimony in these models is achieved by expressing the periodic model parameters in terns of their discrete Fourier transforms. to find a parsimony model in time river discharge time series, "Tesfaye" et al(2007) found from their experience that it is prudent to initially fit a PARMA υ (1,1) model to the data. for more complicated PARMA υ (p,q) models, the periodic ARMA process $\{\tilde{X}_t\}$ with period υ (denoted by PARMA υ(p,q))and the Fourier series representation of the parameters $\phi_t(l), \theta_t(l)$ and σ_t are,

$$X_t - \sum_{j=1}^{p} \phi_t(j) X_{t-j} = \varepsilon_t - \sum_{j=1}^{q} \theta_t(j) \varepsilon_{t-j} \quad (17)$$

$$\theta_t = C_{a0}(l) + \sum_{r=1}^{k} \left\{ C_{ar}(l) COS\left(\frac{2\pi rt}{\upsilon}\right) + S_{ar}(l) SIN\left(\frac{2\pi rt}{\upsilon}\right) \right\} \quad (18)$$

$$\varphi_t = C_{b0}(l) + \sum_{r=1}^{k} \left\{ C_{br}(l) COS\left(\frac{2\pi rt}{\upsilon}\right) + S_{br}(l) SIN\left(\frac{2\pi rt}{\upsilon}\right) \right\} \quad (19)$$

$$\sigma_t = C_{d0} + \sum_{r=1}^{k} \left\{ C_{dr} COS\left(\frac{2\pi rt}{\pi}\right) + S_{dr} SIN\left(\frac{2\pi rt}{\pi}\right) \right\} \quad (20)$$

Where, $X_t = \widetilde{X}_t - \mu_t$ and ε_t = sequence of random variables with mean zero and scale σ_t such that $\{\delta_t = \sigma_t^{-1} \varepsilon_t\}$ is independent and identically distributed. k= total number of harmonics, which is equal to $\upsilon/2$ or υ-1/2 depending on whether υ is even or odd, respectively. The validation of a time series model is tantamount to the application of diagnostic checks to the model residuals to see if they resemble white-noise. To test the white-noise null hypothesis the ljung-box test has used. If the null hypothesis of white-noise residuals is not rejected and if the autocorrelation and partial autocorrelation function of the residuals show no evidence of serial correlation, then we judge the model to adequate.

5. Case Study (KHERSAN River)

Khersan river basin is one of the tributaries of the Karun river catchment. Most parts of this basin are mountainous. Khersan-3 dam site is located in the upper khersan river in south-west of Iran. The mean annual precipitation in the catchment of khersan 3 storage dam is 585 mm. Figure 1 shows the view of khersan river.

Fig. 1: View of Khersan river

stationarity is the first step in time series analysis. So variance and trend stationarity in time series must be probed. Box-Cox transformation used to study variance stationarity. According to the results, Box-Cox transformation shows that data need to change before analyzing to be stationary. Study of Autocorrelation and Partial Autocorrelation functions –after trend analysis– shows non-stationarity and seasonal cycle in data, so differences operators and seasonal differencing need to make them stable.

In domain of frequency Analysis, studied Power Spectrum and periodogram to investigate seasonal and hidden cycles. Based on Fisher Test (T test statistic), data time series have two significant and important seasonal indices in 6 and 12 months (T-test Results not shown). One significant indice in sample data could illustrate a probable definite component. Otherwise, multiple peaks – except those which are created by aliasing – generally shows periodic stochastic seasonal component which is like a multiple seasonal ARIMA process. However, finding the correct order of the model, according to power spectrum is commonly impossible.

According to ACF and PACF functions, 17 common autoregressive models (ARIMA models) fitted to data and modified box-pierce (Ljung-Box) Chi-Square statistic has calculated for each model in 12-24-36-48 lags. Finally, ARIMA(1,0,0)(1,1,1)*12 chosen based on AIC index which defines as,

$$AIC= n \, Ln\,(MSE)+2\,(p+q) \tag{21}$$

Figure 2 shows Residual Analysis for Goodness-fitting of ARIMA (1,0,0)(1,1,1)*12. So time series analysis done for khersan river and the best fitted pattern is selected. Plot of observed and simulated 60 monthly river flows Data for the "Khersan" river –Iran show in figure 3. In this figure, three FSAM, AFAM and F-PARMA models, are simulated for Khersan river time series and results compared with observed data.

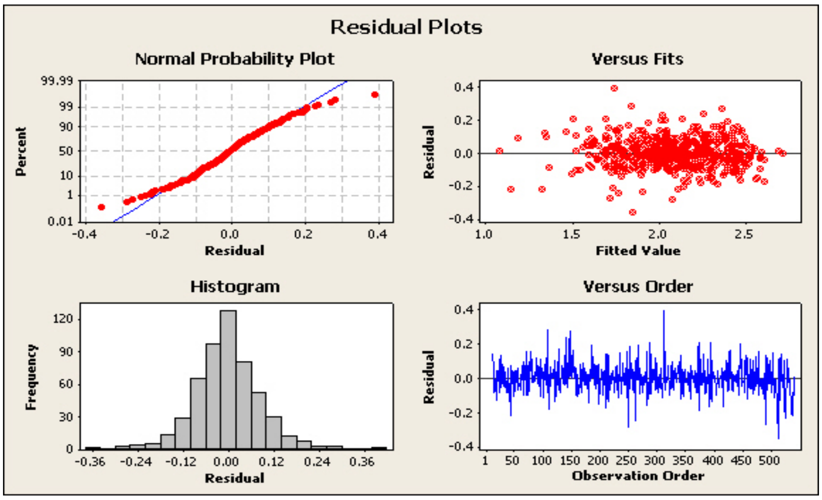

Fig 2: Residual Analysis for Goodness-fitting of ARIMA (1,0,0)(1,1,1)*12

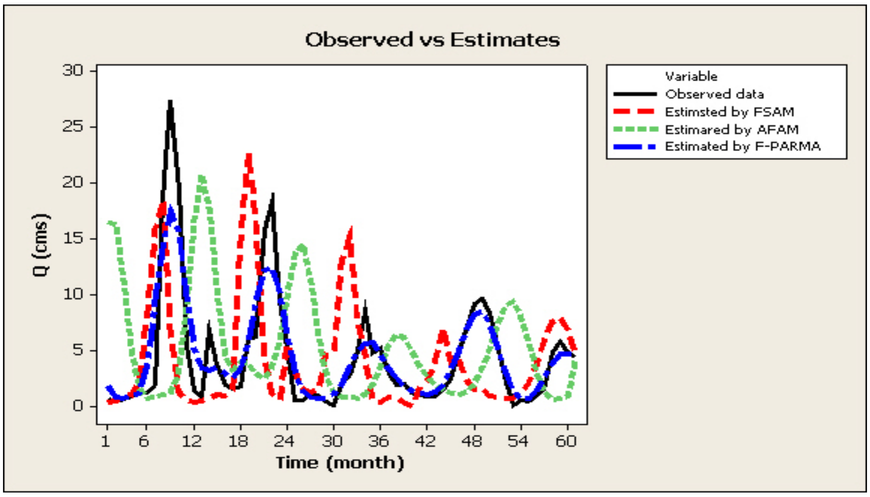

Fig 3: Plot of observed and simulated 60 monthly river flows Data for the "Khersan" river –Iran

6. Conclusion

Results shows ARIMA(1,0,0)(1,1,1)*12 is the best one with minimum AIC index. Additionally, residual analysis results confirm goodness fitting of this model. Comparing observed and forecasted data, illustrates acceptable difference and appropriate model prediction potency.

In this paper, time domain analysis methods studied to analyzing time series and their ability for adapting in results tested to model fitting. Though, according to periodogram and power spectrum, two 6 and 12 months periodic component resulted and both of them were statistically significant, but, just 12 months based fitted model (ARIMA(1,0,0)(1,1,1)*12) had appropriated results.

In this study, using observation information about “Khersan” River along with Fourier, Fourier kalman as well as Fourier PARMA models, time series of river flow has been simulated and applied to predict the future flow. Results showed that every three models were strongly applicable in modeling of dynamic time series. Meanwhile, FSAM model due to application of ARIMA model and elasticity of Fourier equation has been located in a higher position rather than ARIMA linear models. In addition, AFAM model was remarkably able to rapidly reduce the errors and improve the results due to application of kalman filter. Also, PARMA (1,1) model showed a significant accordance with observation information due to application of lower functions. Overall, increasing statistic regarding time series of monthly flows highly develop the applicability of these models in short term predictions in order to be applied in black box modeling.

7. References

[1] Bloomfield, P. Fourier Analysis of Time Series: An Introduction. 2000. John Wiley & Sons, New York, NY, second edition.

[2] Box, G. E. P. and Jenkins G. M. Time Series Analysis: Forecasting and Control, 976. Holden-Day, Oakland, California, revised edition.

[3] Box, G. E. P., Pierce, D. A. and Newbold, P. Distribution of the Residual Autocorrelations in Autoregressive Integrated Moving Average Models, Journal of the American Statistical Association. 1987. 65:1509-1526.

[4] Salas, J. D, Delleur, J.W, Yevjevich, V and Lane, W.L. Applied Modeling of Hydrological Time Series, Water Resource publication.2005.

[5] Vazquez-Amabile, G. G. and Engel, B. A. Fitting of Time Series to Forecast Steamflow and Ground water Using Simulated Data from SWAT. Journal of Hydrologic Engineering, ASCE, 2008. 13(7): 554-562.

[6] Wei, W. S. Effect of Systematic Sampling on ARIMA models, 1987. Theor & Math.

[7] Wei, W. S. Time Series Analysis: Univariate and Multivariate Methods, Ferdowsi University of Mashhad, publication 2008. No.228 (Second Trnslated edition, Niroumand, H. A.).

[8] Ghareaghaji Zare, S. "Time Domain Analysis" Based Modeling of Time Series and Their Application to Prediction of River Discharge, January 21th-23th, 2010. Ahwaz, IRAN (http://www.8irec.ir)

[9] Tesfaye, Anderson, Meerschart, Fourier PARMA models and Their Application to River Flows"-Journal of Hydrologic Engineering- ASCE. 2008.

[10] Salas,J.D. ,J.W.Delleur.,V.Yevjevich.,W.L.Lane 1998."Applied Modelling of Hydrological Time series ", Water Resources Publication.

[11] S.K.Jain, A.Das, D.K.Srivasttava, Application of ANN for reservoir inflow prediction and operation", ASCE journal of Water Research Planning and Management. 2008, 25(5).

[12] Pire, Rostam Afshar, "Application of FSAM and AFAM models in rainfall and discharge prediction" , (M.Sc) thesis , Urmia university-Urmia-IRAN.

[13] Box,G.E.P and Jenkins,G.M (1976)."Time Series Analysis Forecasting and Control",2end ed, Holden day , San Francisco.

[14] S.Ghareaghaji Zare., M.H.Karimi Pashaki and H. sedghi. Fourier Analysis Models and Their Application to River Flow’s Prediction. 2010. Proceeding of The First Internaional Applied Geological Congress, Vol 1. pp:509-513, 26-28 April, Mashhad, Iran.

[15] Wei,W.W."Effect of Systematic Sampling on ARIMA models", 1981. Theor & Math.

[16] N.Rostam Afshar., H.Fahmi and A.R.Pireh."Simulation and Forecasting of River Flow Using Neural Network and a Fourier Series Model. Journal of Iran-Water Resources Research. 2006. 2(1): 36-44.

[17] Y.G. Tesfaye, M.M. Meerschat, and P.L. Anderson. Identification of PARMA models and their application to modeling of river flows. Water Resour. Res, 2006, 42(1), W01419.

[18] Y.G. Tesfaye. Seasonal time series models and their application to modeling of river flows. Doctoral dissertation, Hydrological Sciences. Univ of Nevada, Reno, Nev. 2005, Available at (www.stt.msu.edu/Emcubed/Yonas_thesis.pdf)