

# Evaluation of RBF, GR and FFBP Neural Networks for Prediction of Geometrical Dimensions of Scour Hole Below Ski-Jump Spillway

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**Abstract.** In this paper, Radial Basis Function Network (RBFN), Generalized Regression Network (GRN) and Feed Forward Back Propagation Network (FFBPN) as three different neural network models are employed to predict the principal characteristics of the scour hole geometry shaped downstream of ski-jump spillways, including three distinctive parameters of starting and ending points of the scour hole relative to the spillway bucket lip, as well as the scour hole length. The neural networks consisting of non-dimensional variables are developed. The networks predictions have also been compared with corresponding nonlinear regression equations. The assessments of the models are done in testing data set. The results show that FFBPN and RBFN schemes satisfactorily outperform the regression models and can be used successfully for prediction of the scour hole pattern below ski-jump spillways.

**Keywords:** scour hole dimensions, ski-jump spillway, neural networks, equation.

## 1. Introduction

The local scour process in the plunge pools of ski-jump spillways due to the impact of high energy jet is a serious problem which can lead to the dam structure instability or failure. The majority of earlier studies in this area have focused on developing models to predict the depth of scour hole based on regression approaches, such as, Damle et al. (1966), Mason and Arumugam (1985), and Yildiz and Uzuçek (1994).

Azmathullah et al. (2005) using some experimental data, applied artificial neural networks (ANNs) to estimate the maximum scour hole depth, the location of maximum scour depth from the bucket lip and scour hole width downstream of a ski-jump bucket, the results they gained were satisfactory compared with the conventional statistical regression equations derived from dimensional analysis. Naini et al. (2009) also used neural and neural fuzzy models successfully to predict the scour hole geometry downstream of ski-jump spillway. In this study, the geometrical dimensions of the scour hole in the equilibrium (static) phase below ski-jump spillways (Fig. 1), namely, the same parameters considered by Azmathullah et al. (2005), along with the three unique characteristics of the scour hole, including the starting and ending points of the scour hole relative to the bucket lip and the scour hole length have been predicted using neural network schemes.

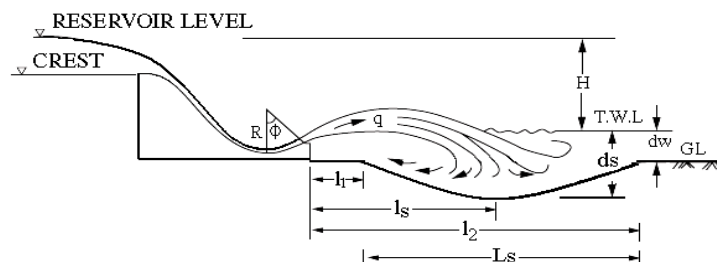


Fig. 1: Geometrical characteristics of scour hole below ski-jump spillway.

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Applicability of three common networks i.e. radial basis function network (RBFN), generalized regression network (GRN) and feed forward back propagation network (FFBPN), is evaluated by employing experimental data for local scour below ski-jump spillways. The neural network models consisting of dimensionless parameters are built, the results are also compared with nonlinear regression equations derived from dimensional analysis using same data set.

## 2. Artificial Neural Networks

Artificial Neural Networks are generally based on our mathematical understanding of the neuro-biological system which consists of simple processing elements called neuron arranged in one or more layer(s). ANNs aim to discover the relationship between parameters through learning process. It creates a mapping between input space (input layer) and output space (output layer) via hidden layer(s). The networks are trained with presented data, during this step the connection weights between the layers are modified until the differences (errors) between predicted and target values are minimized. In this study, two similar and advanced networks of RBFN and GRN from the same family that can be designed and trained in a short time, along with the most commonly used network of FFBPN which takes further time to be built were used.

The RBFN and GRN schemes fit a radial basis function neural network, which is a feed forward, supervised learning network with an input layer, a hidden layer called the radial basis function layer, and an output linear layer. The generalized regression network (GRN) is similar to the radial basis network, but has a slightly different output layer. The FFBPN scheme consists of an input layer, an output layer and one or more hidden layer(s), back propagation learning process with bayesian regularization (BR) algorithm is utilized, as this procedure is robust to high noise level and has good approximation with arbitrary accuracy for training and improving generalization.

## 3. Scour Hole Parameters and Dimensional Analysis

As can be seen from Fig. 1, the geometrical parameters of the scour hole, i.e., the maximum depth of scour measured from tail water level  $d_s$ , the distance of maximum scour depth from the spillway bucket lip  $l_s$ , the distance of starting point of scour hole from the bucket lip  $l_1$ , the distance of ending point of scour hole from the bucket lip  $l_2$ , the scour hole length  $L_s = l_2 - l_1$ , as well as the scour hole width  $w_s$ , can be expressed as a function of fluid and sediment properties involving in plunge pool scour phenomenon, namely, unit discharge of spillway  $q$ , head difference between reservoir and tail water levels  $H$ , radius of the spillway bucket  $R$ , lip angle of the bucket  $\Phi$ , median sediment size of bed  $d_{50}$ , tail water depth  $d_w$ , acceleration due to gravity  $g$ , and densities of water  $\rho_w$  and sediment  $\rho_s$ , which can be written in the following functional form:

$$d_s, l_s, l_1, l_2, L_s, w_s = f(q, H, R, \Phi, d_w, d_{50}, g, \rho_w, \rho_s) \quad (1)$$

In this study, to construct the computational models, the parameters are applied in dimensionless form. To obtain dimensionless parameters, using the  $\Pi$  theorem of Buckingham, Eqn. (1) can be expressed in dimensionless form where the equilibrium scour characteristics are normalized with tail water depth as follows:

$$\frac{d_s}{d_w}, \frac{l_s}{d_w}, \frac{l_1}{d_w}, \frac{l_2}{d_w}, \frac{L_s}{d_w}, \frac{w_s}{d_w} = f\left(\frac{q}{\sqrt{gd_w^3}}, \frac{H}{d_w}, \frac{R}{d_w}, \frac{d_{50}}{d_w}, \frac{\rho_s}{\rho_w}, \Phi\right) \quad (2)$$

In which  $F_0 = [q/(gd_w^3)]^{1/2}$  is the Froude number, the dimensionless parameter of  $\rho_s/\rho_w$  would be constant and can be eliminated from inputs set employed in the modelling.

## 4. Data Set Used

The scour parameters data belonging to two laboratory studies are used. An entire data set containing 96 data are obtained from Momeni Vesalian (2006) and Asadi Saryazdi (1997), respectively, to predict the scour hole geometrical characteristics. Table 1 shows the ranges of various data applied in the present study. The database was divided into two subsets, the first one including 80% of entire samples which selected randomly as the training data set, was used to calibrate the neural models, and the second one including the remaining 20% was used as the testing data set to validate the calibrated models. In order to increase the

accuracy and processing velocity of the networks, all of the input and output data (dimensionless) were normalized and scaled within the range of [0.0, 1.0] before the application.

Table 1: Ranges of data set used.

Sources	No. of Tests	$q(m^3/s/m)$	$H(m)$	$R(m)$	$d_{50}(m)$	$\Phi(rad)$	$d_w(m)$
[6]	32	0.0196-0.0758	1.129-1.404	0.1	0.0002-0.006	0.451	0.06-0.265
[3]	64	0.0204-0.0471	0.2791-0.3827	0.1-0.2	0.008	0.524	0.0286-0.1
Sources (continued)	No. of Tests	$d_s(m)$	$l_s(m)$	$l_2(m)$	$l_1(m)$	$L_s(m)$	$w_s(m)$
[6]	32	0.19-0.44	1.375-2.025	1.625-2.8	0.8-1.4	0.425-1.8	0.26
[3]	64	0.0562-0.3587	0.42-0.82	0.66-1.6	0.1-0.34	0.35-1.35	0.65

## 5. Nonlinear Regression Equations

The regression equations were derived using the same 80% dimensionless data selected randomly. Considering the functional relation (2), following set of equations were deduced for prediction of the geometrical dimensions of the scour hole downstream of ski-jump spillway:

$$\frac{d_s}{d_w} = 3.278(F_0)^{0.702} \left(\frac{H}{d_w}\right)^{0.135} \left(\frac{R}{d_w}\right)^{0.02} \left(\frac{d_{50}}{d_w}\right)^{0.011} (\Phi)^{0.01} \quad (3)$$

$$\frac{l_s}{d_w} = 2.599(F_0)^{0.273} \left(\frac{H}{d_w}\right)^{0.523} \left(\frac{R}{d_w}\right)^{0.034} \left(\frac{d_{50}}{d_w}\right)^{0.01} (\Phi)^{-0.962} \quad (4)$$

$$\frac{l_2}{d_w} = 8.235(F_0)^{0.37} \left(\frac{H}{d_w}\right)^{0.513} \left(\frac{R}{d_w}\right)^{0.01} \left(\frac{d_{50}}{d_w}\right)^{0.01} (\Phi)^{0.01} \quad (5)$$

$$\frac{l_1}{d_w} = 0.322(F_0)^{0.01} \left(\frac{H}{d_w}\right)^{0.857} \left(\frac{R}{d_w}\right)^{-0.01} \left(\frac{d_{50}}{d_w}\right)^{-0.01} (\Phi)^{-1.576} \quad (6)$$

$$\frac{L_s}{d_w} = 11.761(F_0)^{0.656} \left(\frac{H}{d_w}\right)^{0.2} \left(\frac{R}{d_w}\right)^{0.011} \left(\frac{d_{50}}{d_w}\right)^{0.02} (\Phi)^{0.01} \quad (7)$$

$$\frac{w_s}{d_w} = 169.385(F_0)^{0.01} \left(\frac{H}{d_w}\right)^{0.01} \left(\frac{R}{d_w}\right)^{0.096} \left(\frac{d_{50}}{d_w}\right)^{0.891} (\Phi)^{1.675} \quad (8)$$

## 6. Development of Neural Networks

For developing the models MATLAB commercial and computational software was used and coding procedure was employed for gaining the optimum neural networks. To forecast the scour parameters, each kind of the networks was divided into two schemes in which every model can produce three (not six) scour features, the reason for this strategy is to get the highest possible accuracy as the networks bear less pressure for calibrating the weights and biases. Models taking five input parameters  $F_0, H/d_w, d_{50}/d_w, R/d_w, \Phi$  and produce  $d_s/d_w, l_s/d_w, l_2/d_w$  at once considered as Type 1 schemes, and the models taking the same input parameters as above and produce  $l_1/d_w, L_s/d_w, w_s/d_w$  at once are considered as Type 2 schemes, respectively.

The optimized architecture of FFBN models was obtained via the selection of different number of hidden layer(s) and different types of transfer functions as well as various epoch numbers in a trial and error approach. The optimized architecture of RBFN and GRN models was also achieved via the selection of various spread constant values for radial basis function and epoch numbers in a trial and error method. The properties of each neural network model can be seen in Table 2.

## 7. Results and Discussion

Neural networks results are compared to the corresponding nonlinear regression equations results with employing the same data sets. As can be seen from Table 2, performance results of the models are measured in terms of three error criteria, namely, determination coefficient ( $R^2$ ), Root Mean Square Error (RMSE) and Mean Absolute Error (MAE), between the predicted outputs and target (observed) values of the testing data set. It is obvious that in general the forecasting power of FFBN and RBFN models outperform the other schemes in particular regression-based equations, i.e., the  $R^2$  values of FFBN and RBFN schemes are higher than those of regression ones in predicting the geometrical dimensions of the scour hole. In addition, RMSE and MAE values of FFBN and RBFN models are also lower than those of the regression equations.

In contrast with the two other networks, GRN type of network is not satisfactory in outperforming the equations in estimation of the local scour features and indicates deficiency compared with the other schemes. For instance two FFBN and RBFN schemes predict the scour hole depth ( $d_s/d_w$ ) more accurate with  $R^2=0.994$  than the GRN with  $R^2=0.973$ .

In estimation of the starting point location of scour hole ( $l_1/d_w$ ), FFBN outperforms highly satisfactory with  $R^2=0.894$ , RMSE=0.84 and MAE=0.66, respectively, however, the accuracy level for estimation of this parameter is relatively low than the rest of the scour features. When it comes to the scour hole depth ( $d_s/d_w$ ), here RMSE and MAE statistics are lower for RBFN scheme (RMSE=0.25 and MAE=0.21) than FFBN with RMSE=0.29 and MAE=0.22, respectively.

Table 2: Comparison of the predicting performances of the models.

<b>FFBPN</b>	$R^2$	RMSE	MAE	Architecture	Transfer Function
$ds/dw$	0.994	0.29	0.22	5-20-3	tansig – tansig
$ls/dw$	0.992	0.77	0.64		
$l2/dw$	0.991	1.37	1.06		
$l1/dw$	0.894	0.84	0.66		
$Ls/dw$	0.984	1.49	1.31	5-10-3	tansig – logsig
$ws/dw$	0.999	0.15	0.12		
<b>RBFN</b>					
$ds/dw$	0.994	0.25	0.21	5-77-3	radbas – purelin
$ls/dw$	0.990	0.82	0.66		
$l2/dw$	0.990	1.39	1.11		
$l1/dw$	0.843	1.03	0.77		
$Ls/dw$	0.982	1.58	1.22	5-77-3	radbas – purelin
$ws/dw$	0.999	0.05	0.03		
<b>GRN</b>					
$ds/dw$	0.973	0.53	0.44	5-77-3	radbas – purelin
$ls/dw$	0.978	1.19	0.99		
$l2/dw$	0.968	2.55	2.05		
$l1/dw$	0.806	1.07	0.72		
$Ls/dw$	0.945	2.92	2.29	5-77-3	radbas – purelin
$ws/dw$	0.999	0.10	0.05		
<b>Equations</b>					
$ds/dw$	0.970	0.79	0.64		
$ls/dw$	0.975	1.07	0.78		
$l2/dw$	0.952	2.83	1.99		
$l1/dw$	0.800	1.16	0.76		
$Ls/dw$	0.945	2.76	1.98		
$ws/dw$	0.994	0.92	0.81		

The comparative analysis of the results are also illustrated in the form of scatter charts in Figs. 2-4 where FFBN and RBFN schemes forecast the points (yellow and red marks, respectively) in the vicinity of the target values (experimental observations with dark blue marks), while the GRN and the regression-based equations estimate the points (green and purple marks, respectively) in a relatively remote position denoting their higher errors in prediction of the geometrical features of the scour hole.

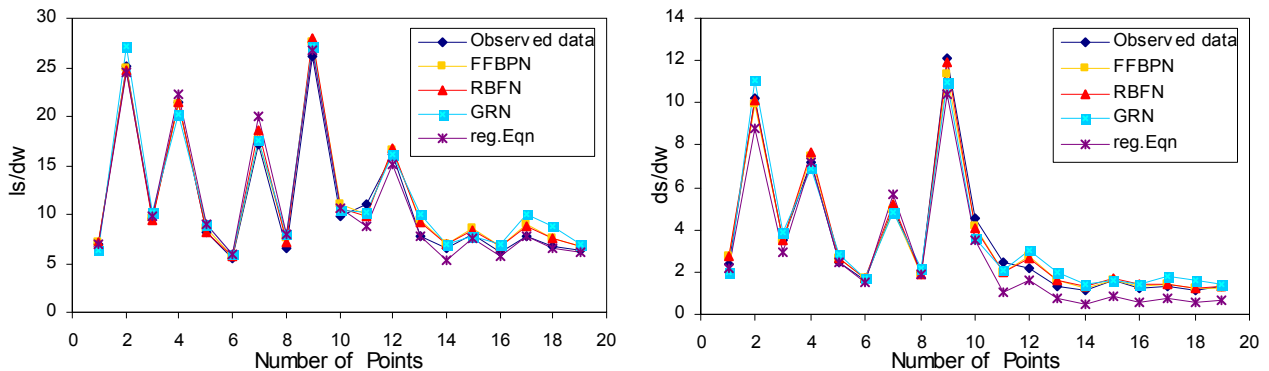


Fig. 2: Performance results of the models for prediction of  $d_s/d_w$  and  $l_s/d_w$  features.

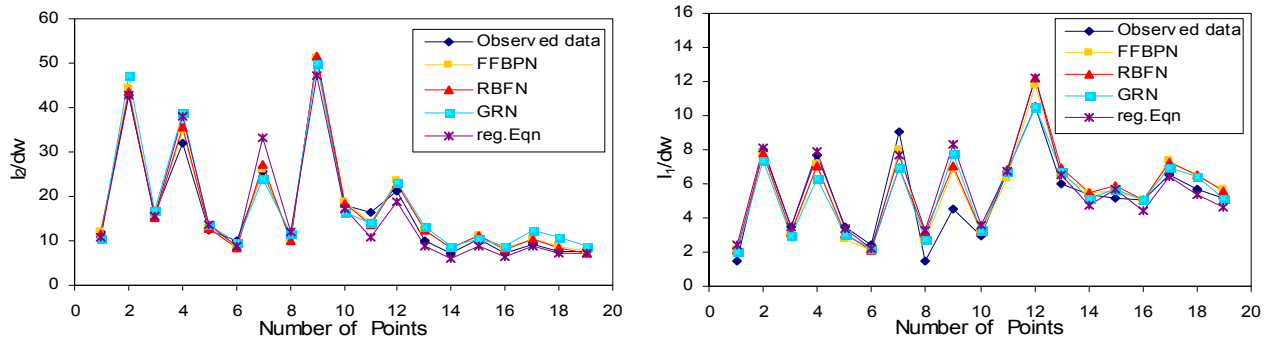


Fig. 3: Performance results of the models for prediction of  $l_1/d_w$  and  $l_2/d_w$  features.

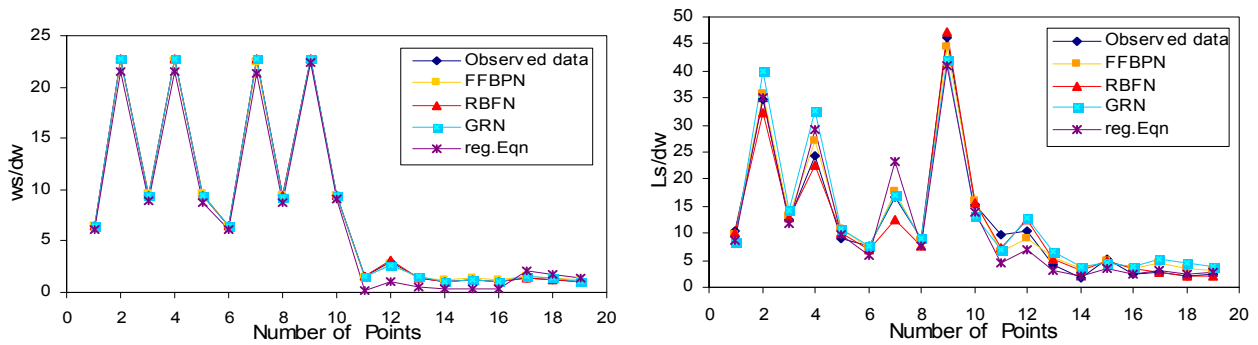


Fig. 4: Performance results of the models for prediction of  $L_s/d_w$  and  $w_s/d_w$  features.

## 8. Conclusions

The three different neural network schemes are evaluated for prediction of the scour pattern due to the erosive jets below ski-jump spillways. In spite of the efficiencies of both FFBPN and RBFN models in estimation of the scour characteristics, FFBPN indicates more reliability and accuracy in terms of the error criteria. Performance of GRN scheme is weak compared to the other neural schemes. A good advantage of using RBFN is that it takes less time to be built than the usual FFBPN scheme. It is concluded that FFBPN and RBFN can be applied successfully for accurate prediction of the scour pattern instead of the equations.

## 9. References

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