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Effect of Autofrettage on Allowable Pressure of Thick-Walled Cylinders

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Abstract. In optimal design of thick-walled cylinders, there are two main objectives to be achieved: increasing its strength-weight ratio and extending its fatigue life. This can be achieved by generating a residual stress field in the cylinder wall prior to use. In the present study, the objective is to present an analytical autofrettage procedure, with the aim of predicting the required autofrettage pressure for various levels of allowable pressure. The results could contribute to a better understanding of the role autofrettage plays in providing strength and performance in pressure vessels. A corresponding FEM validation will be provided. The results reveal three scenarios in the design of thick-walled cylinders. For maximum load carrying capacity, non-autofrettage is suitable when, in service, the whole wall thickness is intended to be yielded. Full autofrettage is suitable when, during subsequent operation, yielding is limited at the inner surface. Optimum autofrettage of the cylinder is suitable if a minimum equivalent stress is required.

Keywords: pressure limits; autofrettage; residual stresses

1. Introduction

Thick-walled cylinders are widely used as critical components in pneumatic and hydraulic systems and as storage and processing vessels. These components require a strict analysis for optimum design to ensure reliable and safe operational performance. The increasing scarcity of materials and higher costs has attracted researchers' attention to the elastic–plastic approach which offers more efficient use of materials.

Autofrettage is a process of generating residual stresses in the wall of a thick-walled cylinder prior to use. A pressure, large enough to cause yielding within the wall, is applied to the inner wall of the cylinder and then removed. Upon release of this pressure, compressive residual circumferential stress is developed to a certain radial depth at the bore. These residual stresses serve to reduce the tensile stresses developed as a result of subsequent application of an operating pressure, thus increasing the load bearing capacity. Using the Tresca yield criteria together, with an autofrettage level parameter, a precise solution for residual stress was developed. The main objective of the paper is to find an optimum autofrettage pressurizing level when the cylinder is subjected to a limiting pressure. In this study, an analytical solution was carried out to investigate the effect of autofrettage on the limit loads of cylinders.

The mechanics and design limitations of autofrettage can be traced to early works by Faupel [1] and then in a later study by Varga [2] by who considered optimum design and showed it to be a function of the diameter ratio and strength of the material. Another comprehensive study on autofrettage of thick cylinders was carried out by Ruilin Zhu and Jinlai Yang [3] who presented an analytic equation for the optimum radius of an elastic–plastic juncture, and the influence of autofrettage on the stress distribution and load-bearing capacity of a cylinder.

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2. Cylinder Subjected to Internal Pressure

For a closed cylinder subjected to an internal pressure, P_i, the radial, circumferential and axial stress distributions are respectively given by Lame's formulation,

$$\sigma_r = \frac{P_i}{k^2 - 1}\left[1 - \frac{r_o^2}{r^2}\right], \qquad \sigma_\theta = \frac{P_i}{k^2 - 1}\left[1 + \frac{r_o^2}{r^2}\right] \qquad \text{and} \qquad \sigma_z = \frac{P_i}{k^2 - 1} \tag{1),(2),(3}$$

2.1 Pressure Limits

Yielding occurs when the Tresca equivalent stress is [8], $\sigma_{Tr} = (\sigma_\theta - \sigma_r) = \sigma_Y$ (4)

Two pressure limits, $P_{Y,i}$ and $P_{Y,o,}$ are considered, to correspond to the internal pressure required to commence yielding of the inner surface, and to cause the wall thickness of cylinder to yield completely. According to Tresca yield strength criterion [4, 9, and 10],

For $P_{Y,i}$, $\frac{p_i}{k^2 - 1}\left(1 + k^2\right) - \frac{p_i}{k^2 - 1}\left(1 - k^2\right) = \sigma_Y$ or $\frac{P_{Tr,Y,i}}{\sigma_Y} = \frac{(k^2 - 1)}{2k^2}$ (5)

As for $P_{Y,o}$, $\frac{2\,p_i}{k^2 - 1} = \sigma_Y$ or $\frac{P_{Tr,Y,o}}{\sigma_Y} = \frac{(k^2 - 1)}{2}$ (6)

Eqs. (5) and (6) are the inner and outer surface pressure limits, respectively.

3. Residual Stresses

If the autofrettage pressure is removed after part of the cylinder thickness has become plastic, a residual stress is locked up in the wall. Assuming that the material is linearly elastic perfectly plastic and during unloading the material follows Hooke's Law, the residual radial, hoop and longitudinal stresses can be obtained from [12],

For the plastic region, $r_i \leq r \leq r_a$

$$\sigma_{r,p,R} = \frac{\sigma_Y}{2}\left\{\left[2\ln\left(\frac{r}{r_a}\right) - 1 + \frac{m^2}{k^2}\right] - \left[2\ln(m+1) - \frac{m^2}{k^2}\right]\left(\frac{1}{k^2 - 1}\right)\left(1 - \frac{r_o^2}{r^2}\right)\right\} \tag{7.a}$$

$$\sigma_{\theta,p,R} = \frac{\sigma_Y}{2}\left\{\left[2 + 2\ln\left(\frac{r}{r_a}\right) - 1 + \frac{m^2}{k^2}\right] - \left[2\ln(m+1) - \frac{m^2}{k^2}\right]\left(\frac{1}{k^2 - 1}\right)\left(1 + \frac{r_o^2}{r^2}\right)\right\} \tag{7.b}$$

$$\sigma_{z,p,R} = \frac{\sigma_Y}{2}\left\{\left[1 + 2\ln\left(\frac{r}{r_a}\right) - 1 + \frac{m^2}{k^2}\right] - \left[2\ln(m+1) - \frac{m^2}{k^2}\right]\left(\frac{1}{k^2 - 1}\right)\right\} \tag{7.c}$$

For the elastic region, $r_a \leq r \leq r_o$, the respective residual stresses are,

$$\sigma_{r,e,R} = \frac{\sigma_Y}{2}\left[1 - \frac{r_o^2}{r^2}\right]\left\{\frac{m^2}{k^2} - \left(1 - \frac{m^2}{k^2} + 2\ln(m)\right)\left(\frac{1}{k^2 - 1}\right)\right\} \tag{8.a}$$

$$\sigma_{\theta,e,R} = \frac{\sigma_Y}{2}\left[1 + \frac{r_o^2}{r^2}\right]\left\{\frac{m^2}{k^2} - \left(1 - \frac{m^2}{k^2} + 2\ln(m)\right)\left(\frac{1}{k^2 - 1}\right)\right\} \tag{8.b}$$

$$\sigma_{z,e,R} = \frac{\sigma_Y}{2}\left\{\frac{m^2}{k^2} - \left(1 - \frac{m^2}{k^2} + 2\ln(m)\right)\left(\frac{1}{k^2 - 1}\right)\right\} \tag{8.c}$$

By substituting $r = r_a$, the residual stresses at the junction radius r_a is obtained,

$$\sigma_{r,R} = \frac{\sigma_Y}{2}\left[1 - \frac{k^2}{m^2}\right]\left\{\frac{m^2}{k^2} - \left(1 - \frac{m^2}{k^2} + 2\ln(m)\right)\left(\frac{1}{k^2 - 1}\right)\right\} \tag{9.a}$$

$$\sigma_{\theta,R} = \frac{\sigma_Y}{2}\left[1 + \frac{k^2}{m^2}\right]\left\{\frac{m^2}{k^2} - \left(1 - \frac{m^2}{k^2} + 2\ln(m)\right)\left(\frac{1}{k^2 - 1}\right)\right\} \tag{9.b}$$

$$\sigma_{z,R} = \frac{\sigma_Y}{2}\left\{\frac{m^2}{k^2} - \left[1 - \frac{m^2}{k^2} + 2\ln(m)\right]\left(\frac{1}{k^2 - 1}\right)\right\} \tag{9.c}$$

On application of the operating pressure the total stress of the autofrettaged cylinder is the summation of the residual stress and the stress due to the operating pressure, i.e.,

$$\sigma_{r,T} = \sigma_r + \sigma_{r,R} \quad , \quad \sigma_{\theta T} = \sigma_\theta + \sigma_{\theta R} \quad \text{and} \quad \sigma_{z,T} = \sigma_z + \sigma_{z,R} \tag{10.(a), (b),(c))}$$

4. Optimum Autofrettage Pressure

During autofrettage, the cylinder has been yielded to the elastic-plastic junction radius, r_a. When the cylinder is subjected to a subsequent internal pressure, the total Tresca equivalent stress is,

$$\sigma_{Tr} = \sigma_Y \frac{k^2}{m^2}\left[\frac{m^2}{k^2} - \left(1 - \frac{m^2}{k^2} + 2ln(m)\right)\frac{1}{k^2 - 1}\right] + \left[\frac{2P_{opr}}{k^2 - 1}\right]\left(\frac{k^2}{m^2}\right) \tag{11}$$

Minimizing the above Tresca stress gives,

$$m = exp(\frac{P_{opr}}{\sigma_Y}) \tag{12}$$

Hence, $$m_{Tr} = exp(n) \tag{13}$$

For a given operating pressure, if the autofrettage pressure P_a is increased, the maximum total equivalent stress on an elastic-plastic junction radius will decrease and then increase again. The value of autofrettage pressure, which gives the minimum value of the maximum equivalent stress on the elastic-plastic junction radius, is the optimum autofrettage pressure $P_{a,opt}$. The corresponding radius is defined as the optimum autofrettage radius, $r_{a,opt}$. The present paper describes the procedure in determining the optimum autofrettage pressure.

The internal pressure to cause yielding to a radial depth of r is,

$$P = \frac{\sigma_Y}{2}\left[1 - \frac{r^2}{r_o^2} + 2ln\,\frac{r}{r_i}\right] \tag{14}$$

From Eq. (13) the optimum autofrettage radius is obtained as,

$$r_{a,opt} = r_i e^n$$

and the optimum autofrettage pressure is calculated as,

$$P_{aopt Tr} = \frac{\sigma_Y}{2}\left[1 - \frac{e^{2n}}{k^2} + 2n\right] \tag{15}$$

Eqs. (14) and (15) show how the optimum autofrettage radius increases with operating pressure.

The relation between the optimum autofrettage pressure and the operating pressure can be obtained from Eqs. (13) and (15).

$$\frac{P_{a,opt,Tr}}{P_{opr,Tr}} = \left[1 + \frac{k^2 - e^{2n}}{2nk^2}\right]$$

which shows that increasing the operating pressure results in an increase in the optimum autofrettage pressure.

5. Autofrettaged Cylinder Subjected to Operating Pressure

For a cylinder treated with partial autofrettage, the internal pressure to cause the inner surface to yield again can be obtained by substituting Eqs. (1), (2) and (8) into Eq. (10). When $r = r_i$, the internal pressure to cause yielding at the inner surface is,

$$P_{opr,Y,i} = \frac{\sigma_Y}{2}\left[2ln\,(m) + 1 - \frac{m^2}{k^2}\right] \tag{16}$$

and when $r = r_o$, by substituting Eqs. (1), (2) and (7) into Eq. (10), the internal pressure to cause the whole wall thickness to yield is:

$$P_{opr,Y,o} = \frac{\sigma_Y}{2}\left[2ln\left(m\right) + k^2 - m^2\right] \tag{17}$$

The values of pressure in Eqs. (16) and (17) are influenced by optimum autofrettage levels which were obtained when an operating pressure was initially known. The internal pressure to cause yielding at the inner surface of a cylinder which is treated with optimum autofrettage pressure is greater than that for a non-treated cylinder. On the other hand, the internal pressure to cause full yielding in a cylinder which has been treated with optimum autofrettage is lower than that which is not treated with autofrettage.

5.1 Fully-Plastic Autofrettaged Cylinder

A special case is when the cylinder is fully plastic during autofrettage, i.e. $r_a = r_o$. Therefore $m = k$ and the equivalent stress at any radius can be obtained from,

$$\sigma_Y = \sigma_Y\left[1 - \frac{2ln\left(k\right)}{k^2 - 1}\left(\frac{r_o^2}{r^2}\right)\right] + \frac{2P_{opr}}{k^2 - 1}\left(\frac{r_o^2}{r^2}\right) \tag{18}$$

Therefore the internal pressure to cause the internal surface and whole thickness to yield is, by substituting $r = r_i$, $r = r_o$ and $m = k$ into Eqs. (16) and (17). The allowable operating pressure of a cylinder treated with full autofrettage, are shown in Fig. 1 which also shows the influence of autofrettage pressure level on the allowable internal pressure of the cylinder. Comparisons of the internal pressure are made between a cylinder which is not treated with autofrettage, treated with optimum autofrettage and full autofrettage.

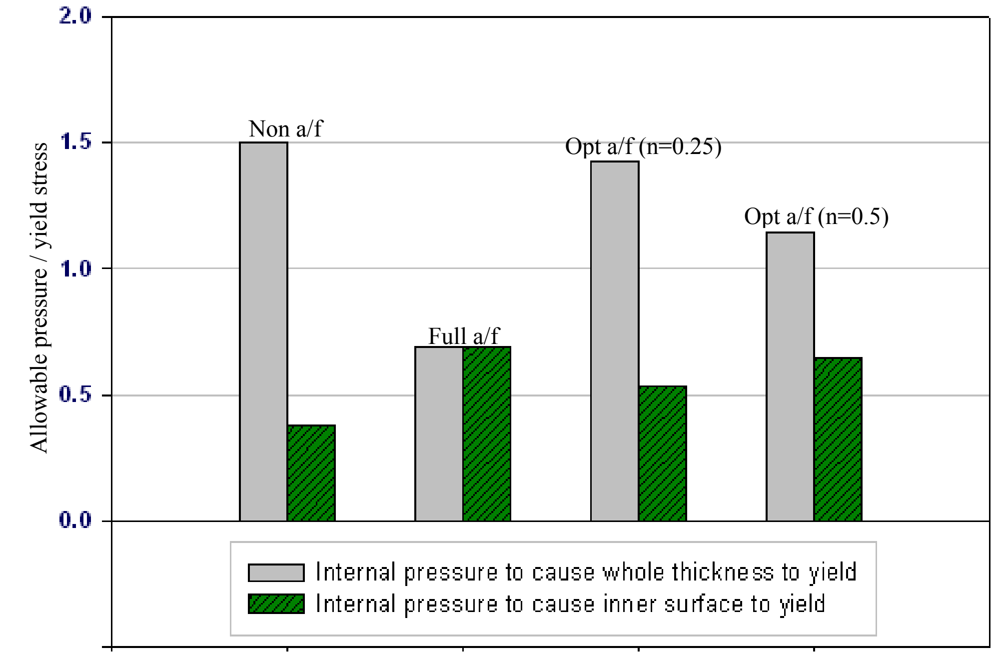

Fig. 1: Effects of different levels of autofrettage on $P_{allowable}$

6. Discussion of Results

A cylinder was chosen to illustrate the procedure of obtaining an optimum autofrettage condition. The material properties and cylinder dimensions are shown in Table 1.

Table 1: Cylinder material properties and dimensions

Elastic modulus, E	203 GPa
Ultimate stress, σ_u	400 MPa
Yield strength, σ_Y	325 MPa
Poisson's ratio, ν	0.3

Inner radius, r_i	0.1 m
Outer radius, r_o	0.2 m
$r_o/r_i = k$	2

An operating pressure of 130 MPa was selected. The above procedure computed an optimum autofrettage pressure of 202 MPa which was applied at the bore of the cylinder. The autofrettage pressure was then released, and the residual stress distributions were evaluated. The cylinder was then reloaded with the internal operating pressure. The analytical procedure gives the optimum results of $P_{a,opt}$=202 MN/m^2 and $r_{a,opt}$=149.2 mm.

7. Conclusions

The effects of autofrettage level parameters on the pressure capacity of cylinders were studied. From the results of this study, the following points may be concluded.

1. The autofrettage process increases the maximum allowable internal pressure.
2. There are three cases of autofrettage in the design of pressurized thick-walled cylinder:
   - Non-autofrettage is suitable if yielding is allowed throughout cylinder wall thickness.
   - Full autofrettage is suitable if yielding is allowed at the inner surface only.
   - Optimum autofrettage case is suitable if the maximum equivalent stress is to be optimized.

8. References

[1] J.H. Faupel. Some Considerations of the Mechanics and Design Limitations of Autofrettage. J. of the Franklin Institute. 1960, 269 (6): 474-489.

[2] L. Varga. Design of Optimum High-Pressure Monobloc Vessels. Int. J. Press. Vessels & Piping. 1991, 48 (1): 93-110.

[3] Ruilin Zhu and Jinlai Yang. Autofrettage of Thick Cylinders. Int. J. Press. Vessels & Piping. 1998, 75 (6): 443-446.

[4] R. Thumser, J.W. Bergmann and M. Vormwald. Residual Stress Fields and Fatigue Analysis of Autofrettage Parts. Int. J. Pressure Vessels & Piping. 2002, 79 (2): 113-117.

[5] Wang Zhiqun. Elastic-Plastic Fracture Analysis of a Thick-Walled Cylinder. Int. J. Pressure Vessels & Piping. 1995, 63: 165–168.

[6] J. Perry and J. Aboudi. Elastic-Plastic Stresses in Thick-Walled Cylinders. ASME J. Pressure Vessel Technology.2003, 125: 248-252.

[7] W. Zhao, R. Seshadri and R.N. Dubey. On Thick-Walled Cylinder under Internal Pressure. ASME J. Pressure Vessel Technology. 2003, 125: 267-273.

[8] R.L. Mott. Applied Strength of Materials. Prentice Hall, 2002.

[9] R. Hill. The Mathematical Theory of Plasticity. Oxford University Press, 1967.

[10] A. Chaaban and N. Barake. Elasto-Plastic Analysis of High Pressure Vessels with Radial Cross Bores. High Pressure Technology. ASME J. Pressure Vessel Technology. 1993, 263: 67–123.

[11] R. Solecki and R. J. Conant. Advanced Mechanics of Materials. Oxford University Press, 2003.

[12] Z. Wuxue, Z. Zichu. An Elastic-Plastic Analysis of Autofrettage Thick-Walled Cylinders. Proc. Int. Conf. on Non-Linear Mechanics. 1985, pp. 663-667.

NOMENCLATURE

P	Pressure
r	Radius
t	Thickness
k	$=r_o/r_i$
m	$=r_a/r_i$
n	$=P_{opr}/\sigma_Y$
σ	normal stress
τ	shear stress
E	elastic modulus

SUBSCRIPTS

i	inner
o	outer
a	autofrettage
r	radial
θ	hoop
z	axial
Y	yield
P	plastic

opt	optimum
opr	operating
max	maximum
min	minimum
Tr	Tresca
R	residual
T	total