

# A Sequential Algorithm for Digital Camera Calibration with Single Images

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**Abstract.** The application of digital camera has greatly promoted the development of close range photogrammetry towards digitalization and automation, and digital images distortion correction becomes a research emphasis in photogrammetric application of digital camera, which has drawn scholars' extensive concern. In view of the problems on the application of existed distortion correction approaches, this paper proposes a sequential algorithm which is flexible and can achieve high precision. By this algorithm, imaging distortion coefficients and elements of interior orientation can be found only by single images without high-precision calibration control field. Experimental results show that this algorithm has fast convergence and gains relatively high precision in actual application.

**Keywords:** camera calibration, digital photogrammetry, distortion correction, image processing

## 1. Introduction

Camera calibration is not only one of the classical problems of photogrammetry and computer vision, but a necessary photogrammetric step to extract reliable 3D metric information from images[1,2]. Distortion is a point position error caused by image points deviating from the ideal position in the design and manufacture process of camera lens. As an important error, it influences the accuracy of image geometric. Most algorithms in photogrammetry and 3D computer vision rely on the pinhole digital camera model because of its simplicity, whereas video optics, especially wide-angle lens, always generate a lot of non-linear distortion[3]. The distortion influences the collinearity condition of photography target imaging, so it leads to many photography sequential algorithms non-convergence and the inaccurate result. And it is also the main cause influencing precision and reliability of photogrammetry based on digital camera.

Hence, digital images distortion correction becomes a research emphasis in photogrammetric application of digital camera, which has drawn scholars' extensive concern. Currently, distortion correction algorithms can be classified roughly into two groups according to the testing conditions: one is computing method based on high precision calibration control field, and the other is estimating method rely on geometric features of shooting targets. A typical example is vanishing point theory. Because the former need to build high precision 3D calibration control field, it's so difficult in operation that not suitable for extensive application. While to the latter, although the operation is so easy, it hardly gets high precision. Based on problems of both methods, this paper proposed a sequential distortion correction algorithm which is flexible and can obtain high precision results. The method is based on perspective transformation and "Luca Lucchese distortion model"[1], estimates the distortion of image points, and finds the distortion parameters of a camera by Sequential computations. Experimental results show that this algorithm has fast convergence and gains relatively high precision in actual application. Therefore, it can be applied in photogrammetry for digital images distortion correction.

## 2. The Algorithm of Camera Calibration

### 2.1. Camera calibration mathematical model

Optical distortion is divided into two kinds of radial distortion and decentring distortion. Radial distortion generally due to the shape defect of the lens, they are only related with the distance from the image point to the principal point, which makes the imaging point  $a$  deviate from accurate location along the radial direction. Decentring distortion generally due to the errors caused by manufacture and installation of the lens, they make imaging points deviate from ideal location along the radial direction and the direction perpendicular to it. General radial and tangential distortion calibration mathematical model for:

A very accurate mathematical model which named as ‘‘Luca Lucchese model’’[2] accounting for both radial and decentring distortions is provided by:

$$\begin{cases} \Delta x = (x - x_0)(k_1 r^2 + k_2 r^4 + k_3 r^6) + p_1(r^2 + 2(x - x_0)^2) + 2p_2(x - x_0)(y - y_0) \\ \Delta y = (y - y_0)(k_1 r^2 + k_2 r^4 + k_3 r^6) + p_2(r^2 + 2(y - y_0)^2) + 2p_1(x - x_0)(y - y_0) \end{cases} \quad (1)$$

Where  $x$  and  $y$  are the photo measuring coordinates of image points,  $\Delta x$  and  $\Delta y$  are the coordinate distortion of image points,  $x_0$  and  $y_0$  are the coordinate of the principal point in the photo coordinate system,  $r$  represents the radial radius of image point, as the distance from the image point to the principal point:  $r = \sqrt{(x - x_0)^2 + (y - y_0)^2}$ ,  $k_1, k_2$  and  $k_3$  are the radial distortion coefficients[3,4],  $p_1$  and  $p_2$  are the decentring distortion coefficients.

By the above formula,  $k_1, k_2$  and  $k_3$  are symmetric distortion coefficients,  $p_1$  and  $p_2$  are non-symmetric decentring distortion coefficients; when obtains five distortion coefficients, we can undertake distortion compensation for arbitrary coordinate of image point. The distribution of distortion error is as follows: the farther the distance from the principal point, the greater the distortion, usually, the principal point is near the center of photo, therefore, the distortion is smaller near the center of photo and the biggest around the edges at the photo; In addition, according to the research, distortion errors dominate in the radial distortion, non-radial distortion errors are about 1/7-1/8 of the radial distortion [5].

## 2.2. Resolution of distortion parameters

To obtain distortion parameters, first step to establish a high-precision two-dimension grid. We can use graphics software to draw a regular grid which displays on the LCD monitor; then use a digital camera to shoot the screen, shown in Fig. 1:

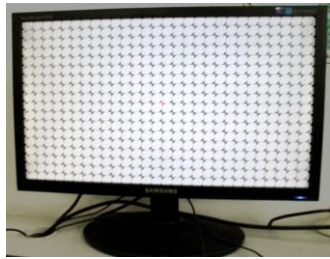


Fig. 1: LCD monitor and plan grid

Taking into account the more grid points, we can first measure a grid point as a template, and then extract the photo coordinate of other grid points automatically through correlation coefficient images matching algorithms. Establishing the object plan coordinate system, which the origin is centre grid point. The formula of object space coordinate about grid points is:

$$x_i = (i - i_0) \times w, \quad y_j = (j - j_0) \times w \quad (2)$$

Here the ranks number of grid are  $(i, j)$ ; the ranks number of central grid are  $(i_0, j_0)$ ; the wide of unit grid is  $w \times w$ , because the algorithm does not need to know the actual size of grid, so the user can set the value of  $w$  arbitrarily.

### a) Projection transformation parameters estimation

In an ideal situation (assuming there is no distortion), object point, image point and central point of projection are strict collinear; image plane and object plane (LCD) are of projection transformation relationship. Projection transformation formula is as follows:

$$a_1 Xx - a_2 x - a_3 y - a_4 + a_8 Xy + X = 0, \quad a_1 Yx - a_5 x - a_6 y - a_7 + a_8 Yy + Y = 0 \quad (3)$$

Where  $X$  and  $Y$  are coordinates of object plan grid points (provided by formula(2)), as well as  $x$  and  $y$  are ideal photo coordinates of corresponding grid points (which have no distortion).

The equation(3) can be rewritten as follows:

$$\begin{cases} x = x' + \Delta x = \frac{(a_4 - X)(a_6 - a_8 Y) - (a_7 - Y)(a_3 - a_8 X)}{(a_5 - a_1 Y)(a_3 - a_8 X) - (a_2 - a_1 X)(a_6 - a_8 Y)} \\ y = y' + \Delta y = \frac{(a_4 - X)(a_5 - a_1 Y) - (a_7 - Y)(a_2 - a_1 X)}{(a_2 - a_1 X)(a_6 - a_8 Y) - (a_5 - a_1 Y)(a_3 - a_8 X)} \end{cases} \quad (4)$$

Here  $\Delta x$  and  $\Delta y$  are distortion,  $x'$  and  $y'$  are image measuring coordinates,  $x$  and  $y$  are ideal image coordinates with no distortion.

This shows that if eight projection transformation coefficients  $a_1 \sim a_8$  according to formula(4), distortion of any grid points can be computed. In fact, the eight projection transformation coefficients cannot be obtained exactly and directly unless by estimation. Method of estimation: choose at least four symmetrical points near the center of image, and turn actual object coordinates and image measuring coordinates into formula(4). As distortions near the center of image are small, we can obtain  $\Delta x = 0$ ,  $\Delta y = 0$  approximatively. Take image coordinates as observed values and then calculate eight projection transformation coefficients through least square adjustment. Adjustment computational processes are as follows:

Linear error equation(4) are as follows:

$$\begin{aligned} vx &= \frac{\partial x}{\partial a_1} \Delta a_1 + \frac{\partial x}{\partial a_2} \Delta a_2 + \frac{\partial x}{\partial a_3} \Delta a_3 + \frac{\partial x}{\partial a_4} \Delta a_4 + \frac{\partial x}{\partial a_5} \Delta a_5 + \frac{\partial x}{\partial a_6} \Delta a_6 + \frac{\partial x}{\partial a_7} \Delta a_7 + \frac{\partial x}{\partial a_8} \Delta a_8 \\ vy &= \frac{\partial y}{\partial a_1} \Delta a_1 + \frac{\partial y}{\partial a_2} \Delta a_2 + \frac{\partial y}{\partial a_3} \Delta a_3 + \frac{\partial y}{\partial a_4} \Delta a_4 + \frac{\partial y}{\partial a_5} \Delta a_5 + \frac{\partial y}{\partial a_6} \Delta a_6 + \frac{\partial y}{\partial a_7} \Delta a_7 + \frac{\partial y}{\partial a_8} \Delta a_8 \end{aligned} \quad (5)$$

Here:

$$\begin{aligned} \frac{\partial x}{\partial a_1} &= \frac{C[(a_3 - a_8 X)Y - (a_6 - a_8 Y)X]}{D^2}, \quad \frac{\partial x}{\partial a_2} = \frac{C(a_6 - a_8 Y)}{D^2}, \quad \frac{\partial x}{\partial a_3} = \frac{D(Y - a_7) - C(a_5 - a_1 Y)}{D^2}, \quad \frac{\partial x}{\partial a_4} = \frac{a_6 - a_8 Y}{D}, \\ \frac{\partial x}{\partial a_5} &= \frac{C(a_8 X - a_3)}{D^2}, \quad \frac{\partial x}{\partial a_6} = \frac{D(a_4 - X) + C(a_2 - a_1 X)}{D^2}, \quad \frac{\partial x}{\partial a_7} = \frac{a_8 X - a_3}{D}, \quad \frac{\partial x}{\partial a_8} = \frac{D[X(a_7 - Y) - Y(a_4 - X)] - C[Y(a_2 - a_7 X) - X(a_5 - a_1 Y)]}{D^2}, \\ \frac{\partial y}{\partial a_1} &= \frac{B[Y(x - a_4) + X(a_7 - Y)] - A[Y(a_3 - a_8 X) - X(a_6 - a_8 Y)]}{B^2}, \quad \frac{\partial y}{\partial a_2} = \frac{B(Y - a_7) - A(a_6 - a_8 Y)}{B^2}, \quad \frac{\partial y}{\partial a_3} = \frac{A \times (a_5 - a_1 Y)}{B^2}, \quad \frac{\partial y}{\partial a_4} = \frac{(a_5 - a_1 Y)}{B}, \\ \frac{\partial y}{\partial a_5} &= \frac{B(a_4 - x) + A(a_3 - a_8 X)}{B^2}, \quad \frac{\partial y}{\partial a_6} = \frac{B[Y(x - a_4) + X(a_7 - Y)] - A[Y(a_3 - a_8 X) - X(a_6 - a_8 Y)]}{B^2}, \quad \frac{\partial y}{\partial a_7} = \frac{a_1 X - a_2}{B}, \quad \frac{\partial y}{\partial a_8} = \frac{A[Y(a_2 - a_1 X) - X(a_5 - a_1 Y)]}{B^2} \end{aligned}$$

Where:

$$\begin{aligned} A &= (a_4 - X)(a_5 - a_1 Y) - (a_7 - Y)(a_2 - a_1 X); \quad B = (a_2 - a_1 X)(a_6 - a_8 Y) - (a_5 - a_1 Y)(a_3 - a_8 X) \\ C &= (a_4 - X)(a_6 - a_8 Y) - (a_7 - Y)(a_3 - a_8 X); \quad D = (a_5 - a_1 Y)(a_3 - a_8 X) - (a_2 - a_1 X)(a_6 - a_8 Y) \end{aligned}$$

Use four points to compute eight projection translation coefficients' initial value according to formula(3).

$$[a_{1(0)} \ a_{2(0)} \ a_{3(0)} \ a_{4(0)} \ a_{5(0)} \ a_{6(0)} \ a_{7(0)} \ a_{8(0)}]$$

Rewrite formula(5) in the following matrix form:

$$\begin{matrix} V \\ n \times 1 \end{matrix} = \begin{matrix} B & X \\ n \times 8 & 8 \times 1 \end{matrix} - \begin{matrix} L \\ n \times 1 \end{matrix} \quad (6)$$

Where:

$$B = \begin{bmatrix} \frac{\partial x_1}{\partial a_1} & \frac{\partial x_1}{\partial a_2} & \frac{\partial x_1}{\partial a_3} & \frac{\partial x_1}{\partial a_4} & \frac{\partial x_1}{\partial a_5} & \frac{\partial x_1}{\partial a_6} & \frac{\partial x_1}{\partial a_7} & \frac{\partial x_1}{\partial a_8} \\ \frac{\partial y_1}{\partial a_1} & \frac{\partial y_1}{\partial a_2} & \frac{\partial y_1}{\partial a_3} & \frac{\partial y_1}{\partial a_4} & \frac{\partial y_1}{\partial a_5} & \frac{\partial y_1}{\partial a_6} & \frac{\partial y_1}{\partial a_7} & \frac{\partial y_1}{\partial a_8} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}_{n \times 8} \quad L = \begin{bmatrix} x_0 - f(x) \\ y_0 - f(y) \\ \dots \end{bmatrix}_{n \times 1}$$

And then, compute least square solutions of eight projection translation coefficients' corrections:

$$X = (B^T B)^{-1} B^T L$$

Finally, compute eight projection translation parameters.

$$[a_1 \ a_2 \ a_3 \ a_4 \ a_5 \ a_6 \ a_7 \ a_8] = [a_{1(0)} \ a_{2(0)} \ a_{3(0)} \ a_{4(0)} \ a_{5(0)} \ a_{6(0)} \ a_{7(0)} \ a_{8(0)}] + X^T$$

## b) Solution of distortion parameters

Substitute eight projection translation coefficients obtained before and all projection coordinates into formula(6), and compute the ideal image coordinates  $x, y$  (assuming no error exit) inversely. Then, estimate distortion of every grid point:

$$\Delta x = x - x', \quad \Delta y = y - y'$$

Substitute distortion into linear equation(1), take distortion residual as observed values, and compute five distortion parameters through least square adjustment. In this process,  $x_0$  and  $y_0$  are unknown but generally small, we can set them to zero beforehand.

### c) Alternate iterative calculation of projection coefficients and distortion coefficients

Obviously, if the computations of projection translation coefficients mentioned above have error, resolution of distortion can hardly be accurate. So iteration of least square adjustment is necessary. Iterate procedure is shown as Fig.2:

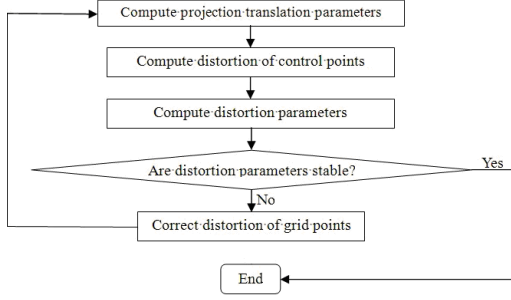


Fig. 2: Alternately iteration procedure of the algorithm

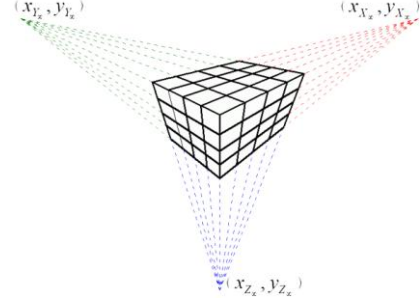


Fig. 3: The diagram of vanishing points

## 2.3. Determination of digital image interior orientation elements

Distortion correction mathematical models need interior orientation elements  $x_0, y_0$  and  $f$ , which can calculate based on vanishing points theory[6]. Vanishing point is the projection of space parallel lines' infinity(Fig.3), that is, the intersection of the parallel lines in the image. It could be thought that the vanishing point, its corresponding spatial infinity and the image principal point meet the collinearity equation.

$$\begin{aligned} x - x_0 &= -f \frac{a_1(X - X_s) + b_1(Y - Y_s) + c_1(Z - Z_s)}{a_3(X - X_s) + b_3(Y - Y_s) + c_3(Z - Z_s)} \\ y - y_0 &= -f \frac{a_2(X - X_s) + b_2(Y - Y_s) + c_2(Z - Z_s)}{a_3(X - X_s) + b_3(Y - Y_s) + c_3(Z - Z_s)} \end{aligned} \quad (7)$$

There will be three vanishing points corresponding with three groups of parallel lines parallel to the  $X, Y, Z$  axis, the coordinates of the points are  $(x_{X_{\infty}}, y_{X_{\infty}})$ ,  $(x_{Y_{\infty}}, y_{Y_{\infty}})$ ,  $(x_{Z_{\infty}}, y_{Z_{\infty}})$  and they meet the formulas as following:

$$\begin{aligned} x_{X_{\infty}} &= x_0 - \lim_{X \rightarrow \infty} f \frac{a_1(X - X_s) + b_1(Y - Y_s) + c_1(Z - Z_s)}{a_3(X - X_s) + b_3(Y - Y_s) + c_3(Z - Z_s)} = x_0 - f \frac{a_1}{a_3} \\ y_{X_{\infty}} &= y_0 - \lim_{X \rightarrow \infty} f \frac{a_2(X - X_s) + b_2(Y - Y_s) + c_2(Z - Z_s)}{a_3(X - X_s) + b_3(Y - Y_s) + c_3(Z - Z_s)} = y_0 - f \frac{a_2}{a_3} \end{aligned} \quad (8)$$

By the same principle:

$$x_{Y_{\infty}} = x_0 - f \frac{b_1}{b_3}, \quad y_{Y_{\infty}} = y_0 - f \frac{b_2}{b_3} \quad (9)$$

$$x_{Z_{\infty}} = x_0 - f \frac{c_1}{c_3}, \quad y_{Z_{\infty}} = y_0 - f \frac{c_2}{c_3} \quad (10)$$

According to formula (8), (9) and (10), interior orientation elements could be calculated:

$$\begin{aligned} x_0 &= [(x_{Z_{\infty}} - x_{X_{\infty}})(y_{Z_{\infty}} - y_{Y_{\infty}})x_{Y_{\infty}} - (y_{Z_{\infty}} - y_{X_{\infty}})(y_{Z_{\infty}} - y_{Y_{\infty}})y_{Y_{\infty}} + (y_{Z_{\infty}} - y_{X_{\infty}})(y_{Z_{\infty}} - y_{Y_{\infty}})y_{X_{\infty}} \\ &\quad - (x_{Z_{\infty}} - x_{Y_{\infty}})(y_{Z_{\infty}} - y_{X_{\infty}})x_{X_{\infty}}] / [(x_{Z_{\infty}} - x_{X_{\infty}})(y_{Z_{\infty}} - y_{Y_{\infty}}) - (x_{Z_{\infty}} - x_{Y_{\infty}})(y_{Z_{\infty}} - y_{X_{\infty}})]; \\ y_0 &= -(x_{Z_{\infty}} - x_{Y_{\infty}})(x_0 - x_{X_{\infty}}) / (y_{Z_{\infty}} - y_{Y_{\infty}}) + y_{X_{\infty}}; \\ f &= \sqrt{-(x_{X_{\infty}} - x_0) * (x_{Y_{\infty}} - x_0) - (y_{X_{\infty}} - y_0)(y_{Y_{\infty}} - y_0)} \end{aligned} \quad (11)$$

According to formula(11), the camera interior orientation elements can be calculated after the calculation of three vanishing points' coordinates. And that, the precision of vanishing points' coordinates directly affect the accuracy of orientation elements. A number of parallel lines should be measured in each direction of  $X, Y$  and  $Z$  to improve the precision, and then the parallel lines will be corrected with the distortion coefficient

calculated by above steps, the intersection of each group parallel lines, that is ,the vanishing point is calculated by least squares adjustment at last.

The interior orientation elements will be substituted into formula(1) after being calculated, and then the distortion coefficients will be re-calculated by above steps, which is a complete process about the calculation of camera distortion coefficients and the interior orientation elements. The substitution will be stopped until the distortion coefficients change small in the process. The results prove that the interior orientation elements have a small influence on the calculation of distortion coefficients.

### 3. Distortion Correction of Original Image

The distortion of original image can be corrected after getting the distortion coefficients and the interior orientation elements. The process of image distortion correction is as follows:

- Generate a blank image M with the same size as original image;
- Correct the distortion of original image one by one pixel. For example, the pixel p with  $(i, j)$  rank number, we should calculate the corrected rank number  $(i', j')$  of p by the distortion correction formula(1) firstly, then get the grayhound value of pixel p and fill it to the pixel with  $(i', j')$  rank number in image M;
- Interpolate the blank pixels. Considering that there must still be several blank pixels in image M after the step 2 of resampling, the inverse distance weight interpolation can be used to interpolate the grayhound value of blank pixels. Search the non-blank pixel in eight directions around the blank pixel Q, set the reciprocals of distances between the searched valid pixels and Q as their weights, and calculate their gray value of weighting average to set it to pixel Q.

The original image and distortion correction image shown in Fig.5 is shot by CanonEos500D digital camera. It's easy to see that line segments in the original image are bending obviously with distortion. The distortion is disappeared by calibration based on above sequential algorithm, which shown as the right image in Fig. 4. As the size of the distortion corrected image should keep the same as the original one, the edge of image will be cut partly. The distortion corrected image can be used in measurement, various photogrammetric calculations and processing.

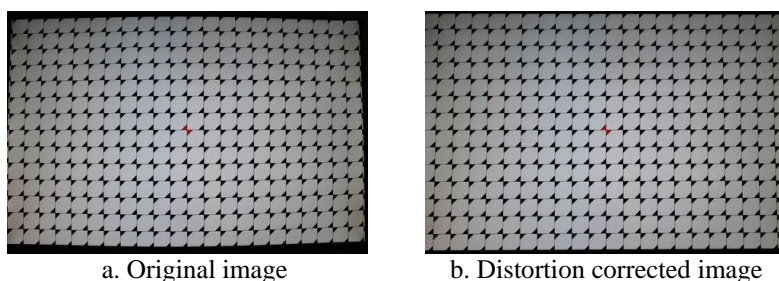


Fig. 4: Distortion correction of original image

## 4. Evaluation of the Algorithm

The evaluation index of the distortion correction algorithm include the convergence of the iterative calculation, the distortion correction residual, and the practical application precision of distortion correction image, etc.

### 4.1. The convergence of the iterative calculation

The algorithm for calculating distortion coefficients presented in this paper is an iterative one, so the convergence is a vital index for the algorithm. In order to checking the convergence of the iteration, we conducted multiple experiments with various types of digital camera, and experimental results revealed that the calculation has fast convergence, and usually converges after about 5 times iteration. Fig. 6 shows the iterative calculation result curve of distortion coefficient of Nikon-D5000 digital camera, where the horizontal axis stands for iteration times, and the vertical axis means value of distortion coefficient parameters.

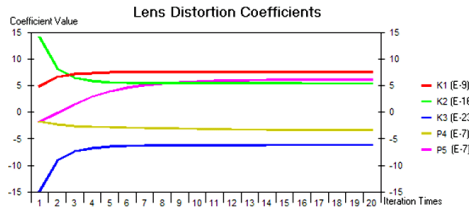


Fig. 5: Iterative calculation result of distortion coefficient (Nikon-D5000)

Tab. 1: Results of interior orientation elements and distortion parameters

Camera Model	Resolution	x0(Pixel)	y0(Pixel)	f(Pixel)	K1	K2	K3	P1	P2
Canon G5	2592×1944	-17.33	33.32	2660.05	2.7531E-08	2.9572E-16	-1.3386E-21	-7.2775E-07	-1.7773E-06
Canon EOS500D	4752×3168	-26.83	45.42	4007.94	1.1171E-08	2.4422E-17	-6.0475E-23	6.9802E-07	-4.3431E-07
Nikon D5000	4288×2848	0.06	-8.79	3394.29	7.5836E-09	5.4106E-16	-6.2315E-23	-3.4302E-07	6.1277E-08

## 4.2. Calibration residual

Distortion correction residual is an important index for measuring the distortion correction results. Using distortion mathematical model to correct points on planar grid above-mentioned, we gain the different value between distortion correction coordinate and the coordinate obtained by perspective transformation inverse calculation based on the space coordinate of control points, and the different value is distortion correction residual. Tab.2 shows distortion residual of several types of digital cameras. Fig. 6 (X axis corresponds with X axis of photo coordinate, Y axis corresponds with Y axis of photo coordinate, Z is distortion correction residual) shows the distribution of corrected distortion (Fig.6 a) and its residual (Fig.6 b) in a whole image, with the 3D model achieved by Kriging interpolation from distortion and its residual of all grid points; The residual distribution is irregular, it can't be fit or eliminated by unified mathematical model.

Tab. 2: Results of distortion residual

Camera Model	Resolution (Pixel)	X Coordinate Residual (Pixel)		Y Coordinate Residual (Pixel)	
		MAX	REMS	MAX	REMS
Canon G5	2592×1944	1.06	0.31	1.26	0.30
Canon EOS500D	4752×3168	2.45	0.43	2.01	0.46
Nikon D5000	4288×2848	1.08	0.30	1.34	0.43

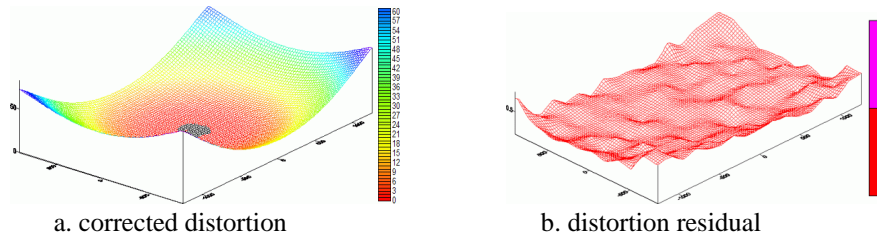


Fig. 6: Distortion residual distribution of a whole image(Canon G5)

## 4.3. Practical application precision

The practical application precision of distortion correction image is the most powerful index for measuring the algorithm. In order to analyze the practical application accuracy, we do experiments in Jiangping River Hydropower station in Hubei province of China by using close-range photogrammetry. This survey area is 200m wide, and 150m in the height. We use Canon-G5 camera with the amplitude of the photo is 2592×1944 pixels and shooting distance about 300m. The entire survey area sets five control points and five check points. Data processing is finished by “EgoInfo Multi-baseline Close Range Photogrammetry System” (Fig. 7) developed by Hohai University. After input interior orientation elements and distortion parameters, corrected images are obtained by the algorithm presented in this paper, and all subsequent photogrammetric processes in the system are based on distortion correction images. The accuracy of check points and control points are shown in Tab.3.



Fig. 7: EgoInfo Multi-baseline Close Range Photogrammetry System

Tab. 3: Results of control point residual and check point residual

Control Point ID	Dx(m)	Dy(m)	Dz(m)	Check Point ID	Dx(m)	Dy(m)	Dz(m)
K2	-0.00414	-0.00764	0.01651	K1	-0.03404	-0.05102	0.05146
K3	-0.00638	-0.00871	0.01826	K5	-0.04460	-0.03781	0.05639
K4	-0.00725	0.02819	0.00330	K7	0.02773	0.04209	0.03987
K6	-0.01381	-0.01761	0.02424	K8	0.02047	0.02908	-0.03763
K10	0.02202	0.01915	-0.00943	K9	0.03841	0.05476	0.04485

The mean square errors of a point are:  $m_x$ : 0.02642m;  $m_y$ : 0.03443m;  $m_z$ : 0.03503m. Relative errors are: 1/11355; 1/8714; 1/8564. The results meet the demand of the design, and are useful in engineering design.

## 5. Conclusion

The application of digital camera has greatly promoted the development of close range photogrammetry towards digitalization and automation. Digital camera metrization of photogrammetry takes distortion as necessary steps is applied widely. In view of the problems on the application of current distortion correction algorithms of digital camera, this paper presents a distortion calibration sequential algorithm based on perspective transformation. By this algorithm, imaging distortion coefficient and elements of interior orientation can be found only by single images without high-precision calibration control field. Compared with other camera calibration approaches, precision achieved by the approach given in this paper, which is based on this algorithm, is in the upper level, and the flexibility and convenience are beyond all comparison. Moreover, this work evaluates the algorithm from three aspects as follows: convergence of the algorithm, residual of distortion correction and precision in practical application. Experimental results show that this algorithm has fast convergence and gains relatively high precision in actual application.

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