

## Optimization Approach to Minimize Energy Consumption in Pasteurized Milk Process

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**Abstract.** Pasteurized milk process is a dairy process that consumes large amount of energy consisting of electricity and fuel. Thermal energy is usually generated by a boiler where energy consumption is ineffective. This paper presents an optimization approach to minimize liquefied petroleum gas (LPG) consumption at the boiler of a pasteurized milk process. Mathematical models of the process have been developed to represent the dynamics behavior of unit operations involved including utilities and validated with actual process data and process requirements. An optimization problem has been formulated and solved to minimize the consumption of LPG and electricity subjecting to the defined pasteurized temperature of 76°C and maximum LPG feed rate to the burner by written programs based on MATLAB software. It has been found that the optimum LPG feed rate is 0.00125 kg/s to achieve the pasteurized temperature. This leads to the LPG and electricity cost saving of 3%. Furthermore, the maximum cost saving can be achieved by the implementation of new optimal LPG consumption rate when the pasteurized temperature is decreased to the minimal pasteurized temperature of 72°C.

**Keywords:** energy cost saving, optimization, pasteurized milk

### 1. Introduction

Pasteurized milk is a dairy product that has a shelf life of 8 to 10 days in an unopened package. Five steps of the pasteurized milk process can be explained. Firstly, the raw milk is reserved in silo tank. Second step is the heat treatment with pasteurization method. The objective of this process is to kill micro-organism in milk under more than 72°C of heating temperature and more than 16 seconds of holding time. Before heating milk, the heat milk is homogenized changing the fat globules into smaller ones for creaming. Next, the pasteurized milk is often cooled to a low temperature approximately 6°C or lower. Lastly, the cooled milk is stored in tanks before going to packing process.

As mentioned above, the pasteurized milk process consumes a large quantity of energy. The electricity is frequently supplied for pumps, homogenizer, and refrigeration unit. The boiler is fuelled by LPG for generating hot water. The expense of electricity and fuel is the second rank of production capital. Currently, the energy and fuel cost is showing increasing tendency, consequently the industrial sector has to find the opportunities in order to improve the energy and fuel consumption.

The consumption behavior energy and fuel can be expressed by the mathematical model and then the integrated models of each unit describe the total consumption of energy for pasteurized milk production. Optimization is a technique to find out the best solution under constraints is applied with mathematical models to determine the optimum condition for processing. This work is aimed at finding out the optimum LPG consumption by optimization method to achieve the maximum efficiency of energy consumption for pasteurized milk production. Furthermore, the mathematical models of process as optimization constraints have been developed and validated with actual plant data and process requirements.

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## 2. Process Model

The principle of mass and energy conversation is carried out to develop mathematical models representing the process behavior of each unit operations in the pasteurized milk process. The pasteurized milk process in Fig. 1 consists of the silo tank, five storage tanks, plate heat exchanger (PHE), holding tube, boiler, cooling tower, ripple plate and three water tanks. Each unit is represented with mathematical model in the form of ordinary differential equations (ODEs).

### 2.1. Assumption

To derive the equations for each unit operations, the following assumptions are adopted.

- Well mixed condition in each unit operation.
- The physical and chemical properties of liquid as density, heat capacity are constant.
- Negligible variation of volume on the both sides of PHE, the heating coil of water tank, the boiler, the ripple plate and the cooling tower.
- Because of well insulation for hot water and iced water piping, the temperature drops at piping surface throughout length of piping are omitted.

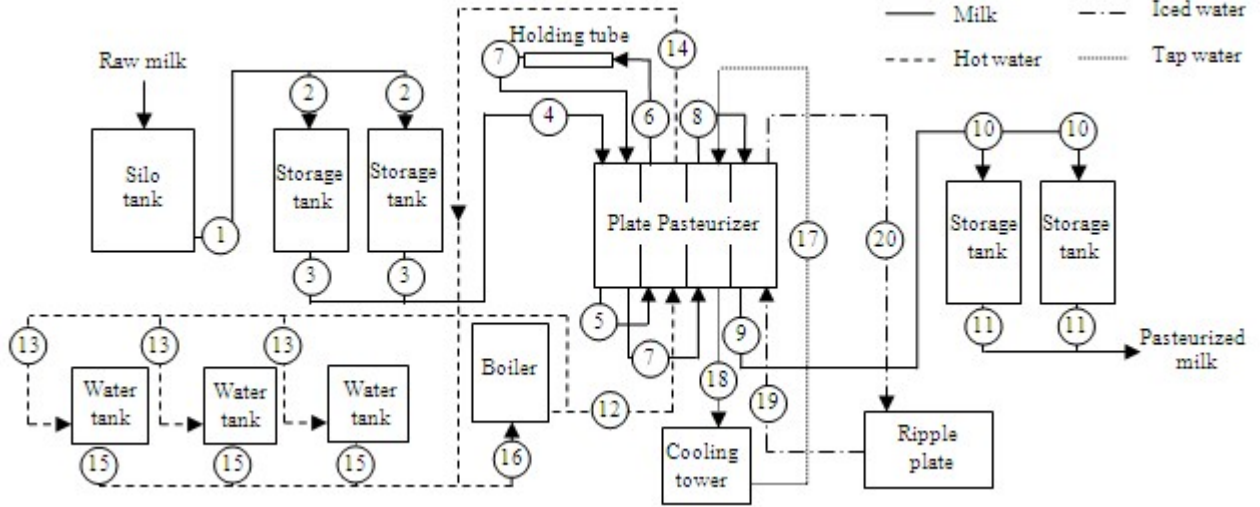


Fig. 1: Schematic diagram of pasteurized milk process.

Mass and energy balance for each unit operations and utilities

#### Silo tank and storage tank

$$\frac{dV_T}{dt} = F_{m,i} - F_{m,o} \quad (1)$$

$$\frac{dT_{m,o}}{dt} = \frac{F_m}{V_T} (T_{m,i} - T_{m,o}) - \frac{U_T A_T}{\rho_m C_{p,m} V_T} (T_{m,o} - T_a) \quad (2)$$

#### PHE

$$\frac{dT_{h,o}}{dt} = \frac{F_h}{V_{PHE}} (T_{h,i} - T_{h,o}) - \frac{U_{PHE} A_{PHE}}{\rho_h C_{p,h} V_{PHE}} \Delta T_{PHE} \quad (3)$$

$$\frac{dT_{c,o}}{dt} = \frac{F_c}{V_{PHE}} (T_{c,i} - T_{c,o}) + \frac{U_{PHE} A_{PHE}}{\rho_c C_{p,c} V_{PHE}} \Delta T_{PHE} \quad (4)$$

#### Water tank

$$\frac{dT_{w,o}}{dt} = \frac{U_{HC} A_{HC} \Delta T_{HC}}{\rho_w C_{p,w} V_{WT}} - \frac{U_{WT} A_{WT}}{\rho_w C_{p,w} V_{WT}} (T_{w,o} - T_a) \quad (5)$$

$$\frac{dT_{hw,o}}{dt} = \frac{F_{hw}}{V_{HC}} (T_{hw,i} - T_{hw,o}) - \frac{U_{HC} A_{HC} \Delta T_{HC}}{\rho_{hw} C_{p,hw} V_{HC}} \quad (6)$$

#### Holding tube

$$T_{m,o} = T_{m,i} - \frac{U_p A_p}{\rho_m C_{p,m} V_p} (T_{m,o} - T_a) \quad (7)$$

#### Boiler

$$\frac{dT_{hw,o}}{dt} = \frac{F_{hw}}{V_B} (T_{hw,i} - T_{hw,o}) + \frac{\dot{m}_f \times LHV}{\rho_{hw} C_{p,hw} V_B} - \frac{U_B A_B}{\rho_{hw} C_{p,hw} V_B} (T_{hw,o} - T_a) \quad (8)$$

#### Cooling tower

$$\frac{dT_{tw,o}}{dt} = \frac{F_{tw} (T_{tw,i} - T_{tw,o}) - L_D F_{cw} T_{tw,i} + \rho_{mw} C_{p,mw} F_{mw} T_{mw}}{V_{CT}} - \frac{(E \rho_{tw} \lambda_v + h_A (T_{tw,i} - T_a))}{\rho_{tw} C_{p,tw} V_{CT}} \quad (9)$$

#### Ripple plate

$$\frac{dT_{iw,o}}{dt} = \frac{F_{iw}}{V_{RP}} (T_{iw,i} - T_{iw,o}) + \frac{CL}{\rho_{iw} C_{p,iw} V_{RP}} - \frac{U_{RP} A_{RP}}{\rho_{iw} C_{p,iw} V_{RP}} (T_{iw,o} - T_a) \quad (10)$$

### 3. Simulation and Model Validation

The geometric characteristics and operating conditions used in the simulation are reported in Table 1. The unknown variables consisting of the overall heat transfer coefficient, the heat transfer coefficient of convection are given by the best fitting method between the simulation results and actual plant data. The actual data are observed by temperature gauge and digital thermometer throughout processing time. The validation results of the mathematical model are reported in Table 2.

Table 1: The geometric characteristics and the simulation operating conditions.

Transfer area of PHE at regenerative stage (m <sup>2</sup> )	1.89	Milk flowrate throughout process (m <sup>3</sup> /s)	4×10 <sup>-4</sup>
Transfer area of PHE at heating stage (m <sup>2</sup> )	1.89	Rate of filling process (m <sup>3</sup> /s)	5×10 <sup>-4</sup>
Transfer area of PHE at pre-cooling stage (m <sup>2</sup> )	1.89	Hot water flowrate in PHE (m <sup>3</sup> /s)	1.60×10 <sup>-3</sup>
Transfer area of PHE at cooling stage (m <sup>2</sup> )	3.99	Water flowrate in PHE (m <sup>3</sup> /s)	4.80×10 <sup>-3</sup>
Plate spacing (m)	0.005	Iced water flowrate in PHE (m <sup>3</sup> /s)	1.92×10 <sup>-3</sup>
Transfer area of heating coil at water tank (m <sup>2</sup> )	0.5067	Hot water flowrate return to the boiler (m <sup>3</sup> /s)	3.20×10 <sup>-3</sup>
Volume of boiler (m <sup>3</sup> )	1.20	Water make up temperature (°C)	27
Volume of water basin at ripple plate (m <sup>3</sup> )	0.50	Ambient air temperature (°C)	30
Volume of water basin at cooling tower (m <sup>3</sup> )	0.05	Rate of circulation water at cooling tower (m <sup>3</sup> /s)	4.80×10 <sup>-3</sup>
Holding tube diameter (m)	3.81×10 <sup>-2</sup>	Drift loss at cooling tower (%)	2
Length of holding tube (m)	12	LPG consumption rate at boiler (kg/s)	1.80×10 <sup>-3</sup>
Rate of pasteurized milk production (kg/day)	1×10 <sup>4</sup>	Refrigeration rate at ripple plate (J/s)	5.20×10 <sup>4</sup>

Table 2: The resulting of variables by best fitting method.

Overall heat transfer coefficient of PHE		Overall heat transfer coefficient of device surface	
- Regenerative stage (W/m <sup>2</sup> ·K)	940	- Holding tube (W/m <sup>2</sup> ·K)	95
- Heating stage (W/m <sup>2</sup> ·K)	940	- Silo tank (W/m <sup>2</sup> ·K)	4
- Pre-cooling stage (W/m <sup>2</sup> ·K)	950	- Storage tank (W/m <sup>2</sup> ·K)	3
- Cooling stage (W/m <sup>2</sup> ·K)	1,000	- Water tank (W/m <sup>2</sup> ·K)	50
Overall heat transfer coefficient of heating coil at water tank		- Boiler (W/m <sup>2</sup> ·K)	200
- Tank 1 (W/m <sup>2</sup> ·K)	600	- Ripple plate (W/m <sup>2</sup> ·K)	20
- Tank 2 (W/m <sup>2</sup> ·K)	490	Heat transfer coefficient of convection between water and air (W/K)	2,000
- Tank 3 (W/m <sup>2</sup> ·K)	380		

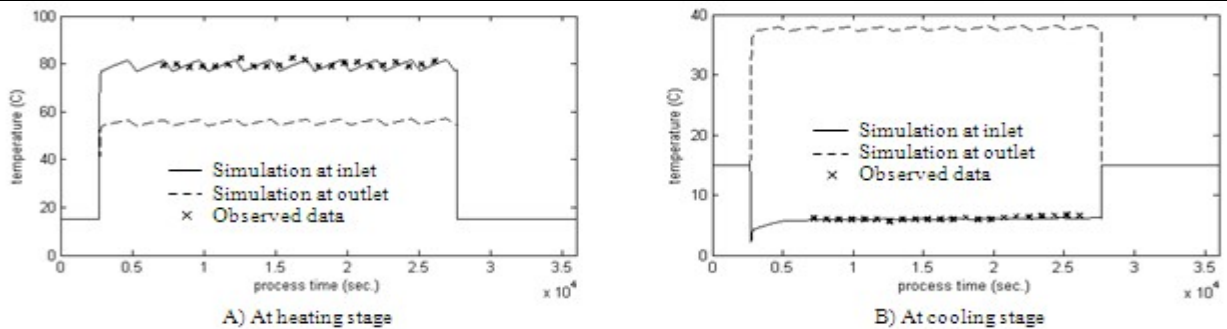


Fig. 2: The temperature profiles on milk side at PHE.

The mathematical models have been successfully formulated to represent the dynamics temperature of the milk and water in unit operations and utilities and the energy consumption for pasteurized milk processing. Due to fluid movement in sequential units, the milk temperature at the outlet of each unit is shifted backward in time by transport delay. After the end of run, the milk temperature is reset to 15°C. The results in Fig. 2 have shown that the fluctuation profiles of milk temperature in PHE are above 76°C at heating stage and below 6°C at cooling stage, because both boiler and ripple plate are subjected to the on-off

controller. The controller maintains the hot and iced water temperature in the range of desired value in order to heat and cool milk at PHE.

## 4. Optimization Problem Formulation

### 4.1. The Objective Function

Due to the fact that the energy expense represents significant production cost, the optimization focuses on the maximum energy utilization in pasteurized milk production. To achieve this, the optimization problem is formed with a single objective problem. The objective is to minimize the annual energy expense related to LPG and electricity cost for pasteurized milk production as the following.

$$\min f = n_w \sum_{t=1}^{36,000} \left( C_{LPG} \dot{m}_f + C_{elect} \frac{(CL + W_e)}{3.6 \times 10^3} \right) \quad (11)$$

$$\text{Subject to process models: (1) to (10)} \quad (12) \quad 4 \leq T_{m,9} \leq 6 \quad (14)$$

$$T_{m,7} \geq 76 \quad (13) \quad 0 \leq \dot{m}_f \leq 0.010 \quad (15)$$

The first term of objective function presents the fuel cost in the boiler. The electricity cost in the second term of objective function contains the electrical consumption in the ripple plate related on the iced water temperature, the pumps, homogenizer, cooling tower and filling machines that is assumed with constant rate during process time. The LPG consumption rate in the boiler is the optimization variable. The optimization constraints represent the process models, the requirement of pasteurized temperature and both lower and upper bound for LPG feeding rate at burner of the boiler.

### 4.2. Optimization Result and Discussion

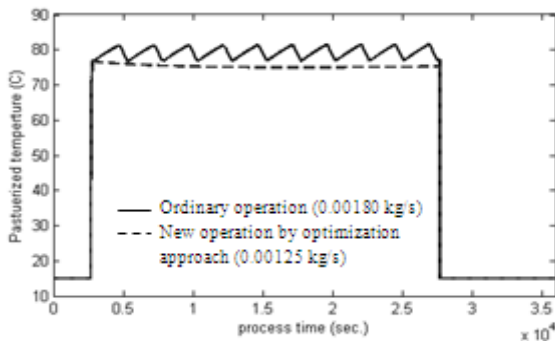


Fig. 3: The comparison with pasteurized temperature profile at holding tube outlet

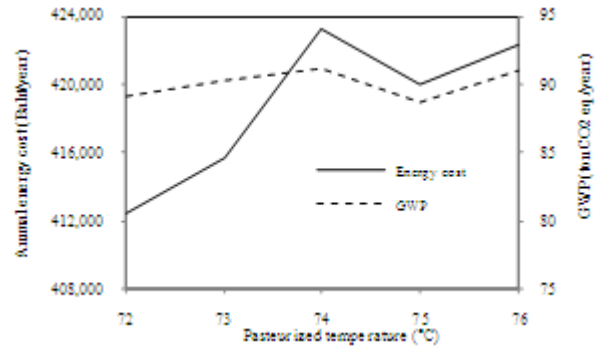


Fig. 4: Annual energy expense and GWP at various pasteurized temperature changing

The optimization problem is classified as constraint nonlinear optimization and solved by sequential quadratic programming (SQP) method in MATLAB. The optimization result shows that the optimum LPG consumption and the annual energy expense based on current operation are 0.00125 kg/s and 419,980 baht/year, respectively. The optimal consumption can control the shape of pasteurized temperature profile that decreases up to reaching a desired value (dash line in Fig. 3), while the pasteurized temperature at the ordinary consumption of 0.00180 kg/s is oscillation profile (solid line in Fig. 3). This optimization result can be implemented by adjusting the LPG feed at the burner which accounts for 11,630 Baht/year of cost saving without any capital investment. Moreover, the reduction of energy consumption, which leads to releasing of greenhouse gas to the atmosphere, can reduce 4.08 tons CO<sub>2</sub> equivalent of global warming potential (GWP) annually.

Moreover, the change of pasteurized temperature coupled with the new optimal LPG consumption is an opportunity to save the energy cost for dairy production. When the pasteurized temperature is changed with the range of 72-76°C, the annual energy expense is illustrated in Fig. 4. The minimal pasteurized temperature has provided the lowest of annual energy expense. Accordingly, the lower bound pasteurized temperature 72°C and corresponding optimal consumption rate of 0.00116 kg/s can save 17,720 Baht/year energy cost and reduce 3.27 tons CO<sub>2</sub> equivalent /year GWP.

## 5. Conclusion

This paper presents the case study of optimization approaches to minimize energy consumption in pasteurized milk process. The optimization problem is solved the optimal LPG consumption to minimize the energy cost. The mathematical models of process validated with the actual data and operational conditions are developed to represent the dynamic behavior of energy consumption and considered as the constraints. The implementation of new optimal LPG consumption still maintains the pasteurized temperature under ordinary operation and gives the energy cost saving with no investment. Moreover, the implementation of new optimal LPG consumption coupled with decreasing of pasteurized temperature nearly to the minimal pasteurized temperature can achieve the maximum energy cost saving.

## 6. Acknowledgements

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## 7. Nomenclature

$A_i$	transferred area of unit i	Subscript	
$C_{\text{Elect}}$	electricity cost	A	water's film at cooling tower
$C_{\text{LPG}}$	LPG cost	a	ambient air
$C_{p,x}$	heat capacity of fluid x	B	boiler
CL	refrigeration load of ripple plate	c	cold fluid at each stage of PHE
E	evaporation rate in cooling tower	CT	cooling tank
$F_x$	volumetric flowrate of fluid x	cw	circulated water at cooling tower
f	annual energy expense	h	hot fluid at each stage of PHE
$h_A$	heat transfer coefficient at water surface	HC	heating coil
$L_D$	mechanical drift loss in cooling tower	hw	hot water
LHV	low heating value of fuel combustion	i	inlet stream
$\dot{m}_f$	fueling rate at boiler	iw	iced water
$n_w$	number of working day	m	milk
$T_{x,j}$	temperature of fluid x at stream j	o	outlet stream
t	time	P	pipng
$U_i$	overall heat transfer coefficient at unit i	mw	make water at cooling tower
$V_i$	fluid volume at unit i	RP	ripple plate
$W_e$	total of electrical power for any equipment	tw	tap water at cooling tower
Greek Letter		w	water
$\rho_x$	density of fluid x	WT	water tank
$\lambda_v$	heat vaporization of water	T	storage tank
$\Delta T_i$	temperature difference among both sides at unit i		

## 8. References

- [1] G. Bylund. *Dairy Processing Handbook*. Tetra Pack Processing System, 1995.
- [2] The Prime Minister's Office, The Notice of Ministry of Public Health. Ministry of Public Health (vol. 266) B.E. 2545 Subject Flavoured Milk. *Royal Thai Government Gazette*. 2003, **120**: 31-39.
- [3] United Nations Environmental Programme (UNEP), *Cleaner Production Assessment in Dairy Processing*. UNEP, 2000.
- [4] Department of Industrial Works (DIW), *Cleaner Technology Codes of Practice (Dairy Industry)*. DIW, 2007.
- [5] P. Kittisupakorn. *Model based control for batch chemical process*. Chula Press, 2008.
- [6] D. E. Seborg, T. F. Edgar, and D. A. Mellichamp. *Process dynamics and control*. John Willey & Sons Inc., 2004.